Wiesław M. Macek

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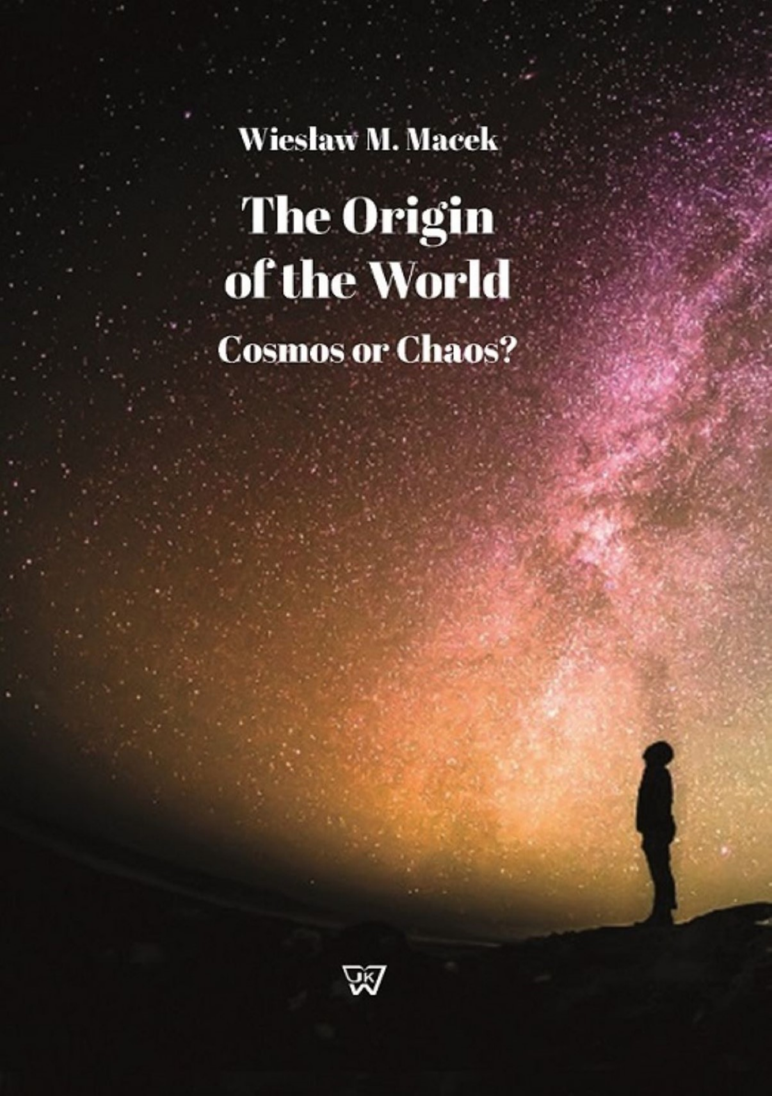
Cosmos or Chaos?



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Preface

Cosmos is a word used to describe the reality of the existing world. The original Greek *κόσμος* can also denote pattern or even order, which suggests that science can facilitate in grasping ultimate explanations of the origin of the Universe. Conversely, chaos (*χάος*) denotes a state of total disorder, although in contemporary chaos theory this term usually refers to a hidden order resulting from nonlinear dynamics. This book argues that together with the fractal geometry of nature these novel concepts used in mathematics and the natural sciences may be useful for a better understanding of the origin of the world.

Part One: *Science* begins with a chapter introducing some basic philosophical and Biblical concepts of the origin of the world. The Standard Model of the Evolution of the Universe based on the Standard Model of Forces is discussed in Chapter 2, together with selected models of the creation of the world based on quantum theory and modern mathematics. Chapter 3 focuses on nonlinear chaotic dynamics and fractals in a search for implications of an unknown nonlinear law related to a hidden order responsible for the creation of the Cosmos at the Planck epoch. Part Two: *Religion* considers the relation between science and religion in Chapter 4, while Chapter 5 is devoted to the question of the meaning of the Universe and the meaning of life. The most salient points of this monograph are briefly summarized in Chapter 6. The Standard Model of elementary interactions and the structure and history of our Universe are illustrated in Appendices A and B, respectively. Finally, subject and authors indexes with glossary terms are provided at the close of the book on page 89.

Wiesław Macek

Warsaw, March 2020

Abstract

This monograph considers the evolution of the Universe according to the standard Big Bang model, quantum models of creation, and the recent theory of nonlinear dynamics, including deterministic chaos, fractals, and multifractals. It shows that by looking for order and harmony in the complex real world surrounding us these modern studies give new insight into the most important philosophical issues beyond classical ontological principles, e.g., by providing a deeper understanding of the age-old philosophical conundrum: *why does something exist instead of nothing?* The book argues that this exciting question is a philosophical basis of matters that influence our lives. In the belief that the concepts of modern science can bridge science and religion, this treatise discusses the consequences of science and religion for the meaning of human life in the vast Universe. In fact, in the mathematical-natural sciences we ought to look for the meaning of the world in the mystery of rationality; for the meaning of the Universe justifies its existence. This scientific view offers meaning and hope to human existence. Therefore, it seems that both science and religion provide important contributions that shape the feelings we experience in the world in which we are immersed.

Keywords: creation, existence, world, reason, science, religion, theology, reality, meaning

1

Introduction

The evolution of the Universe is based on the Big Bang model, which has become the standard scenario in the scientific literature. However, very little is known about the early stages of this evolution, because the required quantum gravity theory is still missing.

For religion, in turn, the creation of the world is usually an important issue, including in the new theology of creation (Macek, 2016). Nevertheless, these two domains of human activity — science and religion — often seem to be in conflict. Some scientists and philosophers, however, have concluded that the aim of science is to explore the Universe created by God; that though science and natural theology have different methods, they have the same subject (Heller, 1996). Despite the fact that the methods of science and religion are different, studies on quantum reality (d'Espagnat, 1983) suggest that they can mutually help each other in our bid to approach Truth (Heller, 2010). In my view, this requires new philosophical concepts based on metaphysics exceeding the classical ontological principles. Moreover, in my own experience science continuously revitalizes my thoughts about God (Macek, 2009, 2010, 2011).

In this book I above all argue that a simple *nonlinear* law, one found within the theory of chaos and (multi-)fractals, may describe a hidden order for the creation of the Universe. Therefore, let me quote words by Henry Miller (1891–1980) that will help us to understand the main issue of this book: *Chaos is the score on which reality is written* (Miller, 1934).

Finally, since I believe that the mathematical natural sciences will permit a better understanding of the meaning of humanity's relation to the Universe, let me cite Michael Heller (born 1936), according to whom *it is in the realm of Meaning that life is worth living* (Heller, 2010). Indeed, both science and religion provide important contributions that shape our grasp of the rich world we experience in our lives.

1.1 Elements of Protology

Protology is a branch of knowledge pertaining to the origin of everything that exists. In Greek *πρώτος*, protos, or *αρχή*, arche (in Latin *principium*) denotes rather the source of existence, and not necessarily the beginning in time. Protology can be juxtaposed to eschatology, which focuses on the final destiny of everything, as the Greek 'eschatos' means 'last'. Eschatology often therefore refers to the end of the world.

Besides the eschatological scenarios in various religions (e.g., the Biblical end of days), physical eschatology predicts that after about 6 billion years the Sun will turn into a red giant, and later even into a white dwarf. Of course, well before then life on the Earth will have become impossible because of global warming. Protology and eschatology are somehow related, if only by the reversibility of time. However, Biblical protology should be interpreted in light of contemporary scientific cosmology.

As a scientific discipline, protology provides the foundations for the theology of creation, which is somewhat independent of the current physical models of the creation of the Universe. In my view, contemporary theological thought should nonetheless open itself to the most important ideas and achievements of modern mathematics and natural sciences.

1.1.1 Philosophical Concepts of the Origin of the World

In order to bridge science and religion, a suitable philosophy is required. Hence one should first call upon the great philosophers, starting with the ancient Greeks, who asked incisive questions about the origin of the existence of the world. In fact, following Empedocles, classical thinkers looked for the basic forms of matter in nature, taking into account all possible primary intuitions. In the terminology of 20^{th} century science, one can even say that they were precursors of those who identified the four states of matter.

It was Xenophanes in particular who noticed that *earth* is essential for every living being, and here one may speak of the solid state. Thales, in turn, considered *water* to be essential for life; this would mean that a liquid should be deemed the primary state of the Universe. But Anaximenes thought that everything originated from *air* which is invisible, and hence the primary Universe was in a gaseous state. It is interesting that the concept of *fire*, as the initial form of everything, was formulated by Heraclitus, and thus may we ascribe to him an early intuition of plasma, the fourth state of matter, now considered to be the main constituent of visible matter in the Universe. Finally, Anaximander speaks about something else, infinite and unlimited, as the origin of everything. We use the Greek word *apeiron* for this.

The most important early contributions to the problem of the origin of the Universe were provided by Plato and Aristotle. According to Plato (ca. 427–347 BC) only eternal ideas really exist, once which always are and never change. Things discernible by the senses are merely shadows of these eternal ideas. The world of true being cannot be apprehended by the senses, but only by an understanding of ideas. In the *Timaeus* Plato explains that the physical word is merely the model of the world of ideas. However, everything in the physical world never really is; rather, it only comes into being. Therefore, the Cosmos itself was not created by any divine intelligence, but by an artist, or the Demiurge (means a craftsman) from the pre-existing formless material (*Chaos*). The Demiurgos created air and water and composed them with the other necessary elements, i.e., fire and earth.

- Therefore, in Plato's philosophy a Demiurge transformed the initial *chaotic* stuff into the ordered *Cosmos*.

The scientific writings of Aristotle (384–322 BC), the influential disciple of Plato, are of course rooted in the Platonic tradition. However, he questioned his master's world of ideas. Aristotle's philosophy originated from fascination with the surrounding world; the world is perfect since it is reasonable. Moreover, contrary to Plato, instead of eternal ideas, Aristotle gazed toward the verification of reality by using the senses, as illustrated by Rafael in his famous *School of Athens*. While Plato's hand indicates the celestial kingdom of ideas, Aristotle's hand points toward terrestrial reality. Hence the latter's metaphysics begins with what is appreciable to our senses. Only by the procedure of abstraction, can we grasp the *ratio* of the existence of being.

It is therefore not surprising that the Aristotelian Universe was spherical, with the Earth surrounded by celestial spheres of planets and stars. Somewhat later, Claudius Ptolemy (ca. 100–170), the mathematician and astronomer, developed the geocentric model of the Solar System based on this conviction. The Ptolemaic planetary system accepted in antiquity lasted until the scientific revolution of the Polish renaissance astronomer Nicolaus Copernicus (Kopernik, 1473–1543), who proposed the heliocentric planetary system. Based on the Greek concept of four basic elements, the Aristotelian world is subject to the attractive forces of gravity resulting from earth and water and other repulsive forces originating from air and fire. The remaining fifth element called *aether*, a divine substance, should fill the gap between the entire spherical Universe and the Earth in the center. The history of astronomy and physics demonstrates again that our senses can often be misleading.

Surprisingly, the foundations of modern science in the seventeenth century, with Newton's laws of motion and universal gravitation, were only possible by rejecting the conclusions arisen from observation in reliance on the senses that any motion requires a force, and proposing action at a distance instead.

Even though the Aristotelian Universe was finite in size, since everything that exists is eternal and basically unchanged, the Universe should always exist. Herein we have the well-known concept of the four causes — namely: the material, formal, efficient, and final causes of every being composed of matter and form. Naturally, the Universe also requires a cause, and since the chain of causes cannot go back to infinity (*regressus in infinitum*), it requires the First Cause (*Prima Causa*). Since the First Cause is the origin of every motion and the world needs motion, the First Cause of the Universe is often called the Prime Mover. In Aristotelian philosophy the first unmoved Mover is God, who does not interfere in the world, nor does the world have influence on Him. Therefore, Aristotle concludes that the world cannot have been created by God, and hence the world has always existed. More precisely, he only explains the structure of the world, and not its origin. Please note that the concept of motion does not correspond to Newtonian motion (where only a change of motion needs a force to accelerate the motion), and the Aristotelian cause is not necessarily related to time. To summarize,

- according to Aristotle the world has always existed, but needed the *eternal* possibly atemporal Prime Mover or the First Cause.

In Late Antiquity a very important contribution to the origin of the Universe came from St. Augustine of Hippo (354–430). St. Augustine tried to understand the question of time, even though he was aware that it would be difficult to grasp the essence of time in general. But St. Augustine had some intuition of true existence, nothingness, and consequently of creation *ex nihilo*. One can say that:

- St. Augustine believed that a Creator (in the fullest sense) was a *Being* from whom the existence (in time) of all things derives (from nothingness in the past to nothingness in the future).

Admittedly, the early Greek philosophers had difficulty in imagining the state of nothingness. Either in the beginning we have something undefined between non-being and being (Plato), or the world has always existed and there is no place for the creation of the Universe (Aristotle). In the medieval period, following Aristotle, Saint Thomas Aquinas (1225–1274) again attributed the First Cause to God and rejected *regressus in infinitum* and even used this argument as the first of five proofs (*Prima Via*) of God's existence. He also argued that the eternal existence of the world does not necessarily contradict its creation. In fact, Aquinas detached the notion of creation from the notion of beginning. He did so by drawing upon Boethius (477–524), for whom eternity is the simultaneous view of the past, the present, and the future: even though the moment of creation happened in the past from our point of view, the everlasting God is able to see the entire history of the Universe from the beginning to the end at once.

1.1.2 Mythological Cosmology

Mythological cosmology is often contrasted with scientific cosmology. The known examples of ancient cosmology based on myths (*theogony*) is the creation by production, e.g., by the most important *deus otiosus*, who is idle, and hence does not require any worship. According to cosmic hierogamy, the earth (*Gaia*) and a personified cosmic force (*Eros*) originated from the mythical *Chaos* and finally Cronus and Rhea gave birth to the Greek god *Zeus*. Following the Greek *Titanomachy* of the competing *Titans*, the Babylonian myths preferred creation resulting from the battle between the gods (e.g., *Enuma Elish*). In the end, in a more subtle ancient Egyptian ('monotheistic') religion, the intellectual god *Ptah* creates with 'heart and word', suggesting that the fate of the man arising from the tears of the god *Re* is torment. Even though the Biblical authors knew these cosmogonies, they questioned all mythical theogonies popular in the cultures of the surrounding world.

1.1.3 Biblical Concepts of Creation

The Bible was of course written over many centuries and the Biblical authors reflect various cultures. Moreover, the question of the origin of the world is here merely a background to the history of salvation. After all, since contemporary science was founded not until the seventeen century, it would be difficult to find scientific descriptions in the Bible.

But it seems that the main discovery of the Bible is that the created world differs from God. We cannot be afraid of nature since God Himself calls us to make the Earth the subject of men (Genesis 1: 28). Therefore, in the Biblical descriptions the stars, the Sun, and the Moon are simply celestial bodies created by God (Genesis 1, 14–18), in contrast to the ancient Greek philosophers somewhat related to Greek mythology, who (as did Aristotle) attributed them certain properties of nearly divine perfectness. On the contrary, it seems that the debunking of myths, together with their etiological interpretation (in Greek $\alpha\iota\tau\acute{\iota}\alpha$, aitija), may well be considered the origin of early science (Heller, 1996).

Figure 1.1 The first verse in the original Hebrew Bible.

The main Biblical message is that everything (both the spiritual and material world) was created by God, but the authors do not intend to explain how it really happened. The first sentence of the Bible (Genesis 1,1) is:

In the beginning, God created the heaven and the earth.

By the way, the Book of Genesis is not the oldest Biblical scripture, and the concept of creation had been developing over centuries. This first verse in the original Hebrew reads, Figure 1.1: *Bereshit bara Elohim et hashamayim ve'et ha'aretz*. Following exegetic interpretations this expresses the fundamental truth about the origin of everything from God, who is the Lord of the whole world. Obviously, the heaven and earth mean everything that exists, i.e., the Universe.

Even though the interpretation of this important announcement of the beginning (in Hebrew, *bereshit*) could sometimes be related to the temporal beginning, by no means can this be inferred from the text. In the Old Testament, God (*Yahweh*) is the most powerful Lord (*Adonai*) of the chosen nation from the early history of *henotheism* (*henos theos* means 'one god') and ultimately the only God, as the word *Elohim* signifies in the strongly *monotheistic* Biblical interpretation. The other gods simply do not exist. Therefore, the Bible is very critical toward mythical cosmology, which resulted from the personification of various cosmic elements.

Consequently, in the Hebrew Bible the masculine verb *bara* (used in Genesis 1, verse 1; Genesis 2: verses 3 and 4) denotes the act of creation that can only be attributed to God, who created everything out of NOTHING (in Latin *ex nihilo*)[1]. Formless empty matter (*chaos*, in Biblical Hebrew *tohu wa-bohu*) over the surface of the deep primordial water, or bottomless pit, hebr. *tehom*

[1] The term will be coined later in the period of the Babylonian captivity, cf. 2 Maccabees 7: 28.

(Genesis 1: 2) are merely mythical elements taken as images of nothingness[2]. In the book of Genesis we have the following two images that were everywhere in the beginning: (1) water (Genesis 1: 1–2,4a) according to the (*Elochist*) *Priestly* tradition (P, dated VI/V century BC) or alternatively (2) the earth as a desert (Genesis 2: 4b–24) preferred by the Yahwist tradition of the Biblical source (J, X/IX century BC). These two models of the origin of the world can be called *aquatic* (from the Latin *aqua*, water) and *terric* (from the Latin *terra*, earth) cosmology, respectively.

The act of the creation of matter begins with the creation of light separated from darkness (the first day), followed by the creation of the heavens separated from the water (the second day), the solid earth and the sees (the third day), and heavenly bodies, the Sun, the Moon, and the stars (the fourth day). The living creatures were created on the fifth day, and finally man in His own image on the sixth day (Genesis 1:3–2:3). The days of creation are beautifully illustrated in Figure 1.2, taken from the Nuremberg Chronicles (*Die Schedelsche Weltchronik*, 1493). However, St. Augustine argued that the period of creation in just six days cannot be taken literally. He interpreted heuristically the Biblical description in the Book of Genesis rather as a logical framework. For example, referring to the passage in Sirach 18:1 using *creavit omnia simul* (created all things at once)[3], God should have created everything in the Universe simultaneously, at least 'in seeds'. Even though, everything that was created during the six days is good or even very good, the seventh day is needed to finalize creation. It is evident that the digit seven symbolizes perfectness, and that God's resting on the seventh day is a good example for society.

The Biblical authors were naturally influenced by Greek philosophy, including the philosophical terms of *arche* (e.g., the elements: earth and water). However, since Divine revelation should be based on truth that is free of myths, criticism of the myths is therefore continued in the New Testament. Saint Paul called the myths 'fables' (1 Timothy 1:4; 2 Timothy 4:4; Titus 1:4), and Saint Peter (2 Peter 1:16) spoke clearly about devised fables. The Greek philosophers, e.g., Heraclitus (535–475 BC), coined the term *Logos* (the Word)

[2] Some allusions to the dragons in the water, probably borrowed from Babylonian cosmology can hardly be found in Psalm 74: 13 ('hast scattered thine enemies with thy strong arms', Psalm 89, 10); cf. Ezekiel 28: 12 12–19 describing the fall of the perfect creation, perhaps the only Biblical place about angels (cherubs), where one reads: *So I drove you in disgrace from the mount of God, and I expelled you, guardian cherub, from among the fiery stones.*

[3] In the original Hebrew version one rather reads: *created all thing together (in general or even 'without exceptions').*

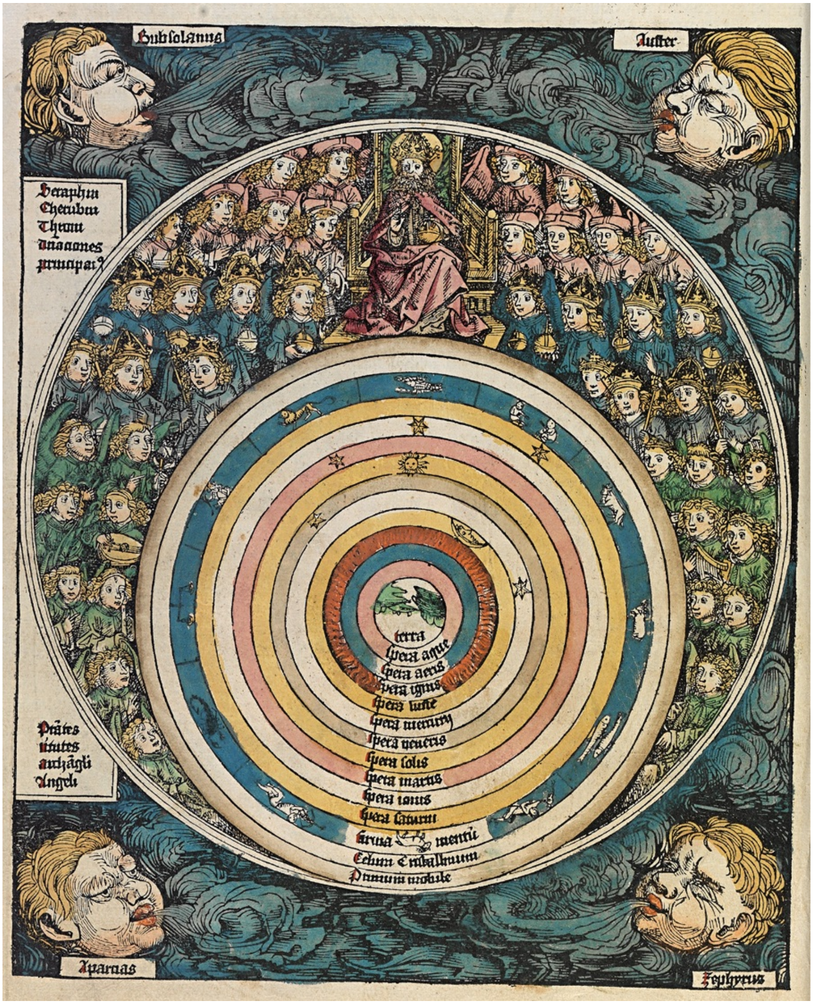

Figure 1.2 The cosmography of the cration according to Genesis (1493).

to denote a principle of order and knowledge. Other ancient philosophers used the term in different ways, e.g., as discourse (the sophists), a reasonable discourse (Aristotle), and the principle of the Universe (the stoics). It seems that Philo of Alexandria (ca. 20 BC – 50 AD), a Hellenistic Jewish philosopher, adopted this term into Jewish philosophy.

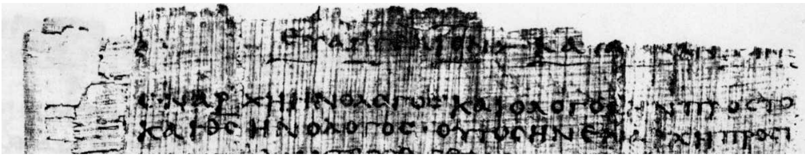

Figure 1.3 The prologue of St. John Evangelist, written on 66 Bodmer's papyrus (ca. AD 200), Geneva.

Early Christians were convinced that the order in nature is a sign of the Divine origin of the world. In addition, the relation of Wisdom with Logos should result in the comprehensibility of the physical world (Book of Wisdom, 7). Finally, St. John the Evangelist (ca. 15–100 AD) identifies the Christian Logos as the creator of the Universe, and further with the incarnate Logos, Jesus Christ. Therefore, the origin of everything is not any of the classical elements (earth, water, air, or fire), but the Word (*Logos*). Consequently, the first sentence of Gospel of John (1, 1) asserts that *in the beginning was the Word*. In the original Greek this reads, Figure 1.3:

Eν ἀρχῇ ἦν ο λόγος.

1.2 Toward a Science of Creation

In this book I shall consider the origin of the Universe in the light of modern science, including quantum models of creation, recent theories of nonlinear dynamics, deterministic chaos, and fractals.

I hope that these studies will offer us new insight into the most important philosophical issues exceeding the classical ontological principles, e.g., by providing a deeper understanding of the philosophical dilemma: *Why does something exist instead of nothing?* (Leibniz, 1714). Here Gottfried Wilhelm von Leibniz (1646–1716) referred at the close of his life to the old-age question of Parmenides (ca. 515–475 BC). I argue that this dramatic problem is the philosophical basis of the problems that shape our lives.

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PART ONE

SCIENCE

2

The Universe in Modern Science

The classical concept of absolute space was created in antiquity by Euclid of Alexandria (ca. 325–265 BC). Admittedly, Euclidean geometry still well describes physical space on a macroscopic scale, but this geometry is certainly limited on the very small scale of the dimensions of atoms ($\sim 10^{-10}$ m), where quantum effects begin to play an important role. On the other hand, one cannot expect that flat geometry is suitable in the case of strong gravitational fields, e.g., in the vicinity of black holes with masses comparable to (or greater than) the mass of the Sun, but with dimensions of only a few tens of kilometers.

One should mention that the philosophical concept of space and time originated from aprioric geometry proposed by the German philosopher Immanuel Kant (1724–1804). Even though he was aware that space and time are rooted in empirical reality, because a human being is only able to perceive dynamical changes using the categories of our mind, he attributed these aprioric categories with ideal transcendental forms. Isaac Newton (1643–1727) treated space and time separately. For him an absolute and empty space was like a 'bag' for material points. Since Newton looked at the world in a mechanistic way, he considered these material points as moving in time on trajectories, resulting from gravitation forces according to the laws of dynamics[1]. In this approach, it is even possible to imagine empty space without any matter. In contrast, Gotfried W. Leibniz (mentioned on page 10) considered space and time to be relations between events and things. Therefore, according to Leibniz, God created the Universe together with space and time. Hence, one cannot imagine space without matter — and thus we need ponder the concept of spacetime.

[1] Pierre Simon de Laplace (1749–1827) was convinced that if we knew the initial positions of all the bodies, then using Newton's laws of dynamics one should be able to predict any event for all time and any place in absolute space.

2.1 The Geometry of Spacetime

The modern concept of spacetime as a set of events localized in space and time was proposed by Albert Einstein (1879–1955), to whom we owe the foundation of a new theory of physics. Namely, it turned out that the special relativity he proposed in 1905 properly describes the dynamics of material bodies, even those moving at velocities comparable (or equal) to the speed of light. Einstein made avail of a concept for the combination of three-dimensional Euclidean space and time (as the fourth dimension), as proposed by his teacher Hermann Minkowski (1864–1909). Points in Minkowski space are determined by three components corresponding to classical physical space and one component corresponding to time. Because time is treated differently than three-dimensional space, Minkowski spacetime differs from four-dimensional Euclidean space (in signs of pseudo-Euclidean metrics).

In the theory of special relativity, according to the principle of relativity, the laws of dynamics should be the same in all inertial frames of reference, e.g., moving with a constant velocity along a straight line (or at rest). In addition, the speed of light is constant c (independent of the frame of reference), as the maximal velocity of propagation of any physical signal in the Universe. Hence, owing to the transformation formulated by the Dutch physicist Hendrik Lorentz (1853–1928), one should have contraction of distances and dilation of time in the frame of reference moving relative to the observer. In particular, the concept of simultaneity becomes relative, i.e., depends on the frame of reference. Moreover, any mass m is equivalent to energy E, according to Einstein's celebrated formula: $E = mc^2$.

However, this veritable revolution in understanding space, time, and matter was achieved only a century ago (1916) owing to the foundation of general relativity. This theory can be applied even in the case of strong gravitational fields. Again, in accord with the general principle of relativity, physical laws should be independent of the observer, even in the case of a noninertial frame of references (i.e., moving with acceleration). Surprisingly, one should conclude that spacetime and matter cannot be independent. According to general relativity, gravitation is revealed by the curvature of local spacetime. as schematically shown in Figure 2.1. Instead of the flat four-dimensional Minkowski spacetime we should involve a non-Euclidean spacetime with positive (elliptic type) or negative (hyperbolic) curvatures, respectively, as formulated by Georg F. B. Riemann (1826–1866). Obviously, Minkowski geometry (corresponding to four-dimensional Euclidean pseudo-space) is only a special case of Riemannian geometry. Therefore, we may briefly state that mass (energy) tells spacetime geometry about its curvature, but curved spacetime tells the mass how to move.

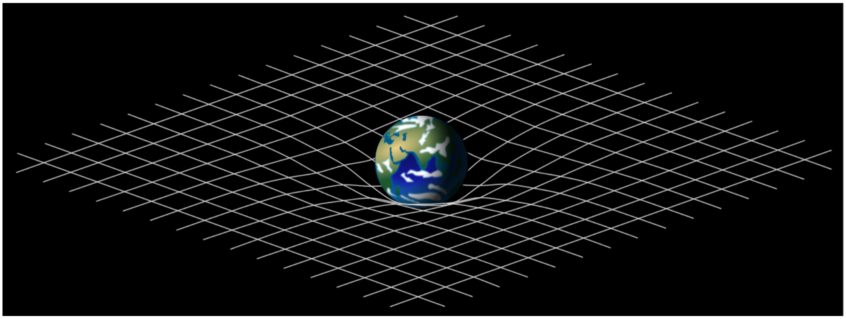

Figure 2.1 Gravitation and geometry.

2.2 The Standard Model of the Evolution of the Universe

Owing to twentieth-century studies on general relativity, physical cosmology has developed into the Standard Cosmological Model[2] (Peebles, 1993). The rough features of the evolution of the Universe that originated from an unknown initial state as predicted by the well-known Big Bang scenario are now confirmed by a wealth of observations acquired from various missions in space.

2.2.1 The Big Bang Model

According to the Big Bang model, the Universe expanded from an extremely dense and hot state and continues to expand today. A common analogy explains that space itself is expanding, carrying galaxies with it, like spots on an inflating balloon. The depiction in Figure 2.2 is an artist's conception illustrating the expansion of a portion of a simple model of the flat Universe with two space dimensions[3], see Figure 2.2.

A more realistic representation of the Universe's evolution is schematically shown in Figure 2.3. Based on the best available measurements of the

[2] In 2019 James E. Peebles (with Michel Mayor and Didier Queloz) was awarded the Nobel Prize in Physics for contributions to our understanding of the evolution of the Universe and Earth's place in the cosmos.
[3] Taken from http://en.wikipedia.org/wiki/Big_Bang

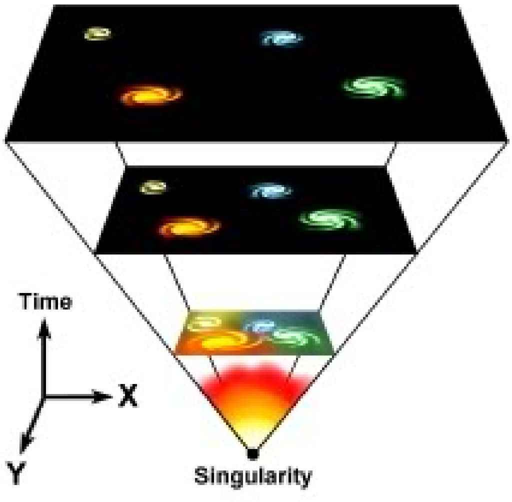

Figure 2.2 The expanding Universe.

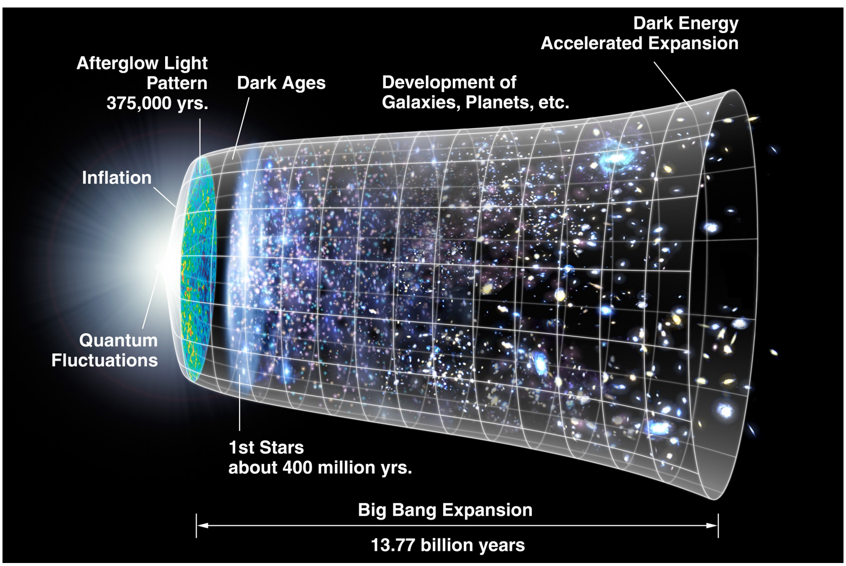

Figure 2.3 Schematic of the evolution of the Universe, credit: NASA / WMAP Science Team.

Wilkinson Microwave Anisotropy Probe (WMAP) operating from 2001 to 2010, the original state of the Universe began around 13.8 billion years ago, when the Big Bang occurred. This was possibly followed by 'inflation', producing a burst of exponential growth in the size of the Universe. The far left depicts the earliest moment we can now probe: size is depicted by the vertical extent of the grid in this graphic.

The Planck mission launched in 2009 (deactivated in 2013) has become the most important source of information about the early Universe by providing unique data at microwave and infra-red frequencies with high sensitivity and small angular resolution. The structure and history of our world based on the Planck's achievements showing the respective scales of the galaxy and the Solar System are illustrated in Appendix B.

As seen in Figure 2.3, the first microsecond, consisting of electroweak, quark, and hadron epochs, together with the lepton epoch (until 3 minutes of its existence) was decisive for further evolution, leading to the nucleosynthesis of helium from hydrogen (cf. Hawking, 1988). Only after 70 thousand years was light separated from matter. The afterglow light seen by WMAP was emitted about 400 thousand years after the beginning (when the electrons and nucleons were combined into atoms, mainly hydrogen) and has traversed the Universe largely unimpeded since then. The conditions of earlier times are imprinted on this light; it also forms a backlight for later developments of the Universe. The first stars appeared about 400 million years later. The Planck data in Figure B.3 suggest that the Dark Ages (before the first star appeared) ended somewhat later, i.e, 550 million years after the Big Bang. This mission has also provided a new catalog of more than 1500 clusters of galaxies observed in the Universe. More than 400 of these galaxy clusters have large masses ranging between 100 to 1000 times that of our Milky Way galaxy.

After the formation of galaxies, and finally, our solar system, about 5 billion years ago, for the next several billion years the expansion of the Universe gradually slowed as the matter in the Universe pulled on itself by gravity. One can ask whether the present expansion will continue forever or if it might eventually stop, thereby allowing a subsequent contraction. Even though we cannot give a definitive answer to this question, recently it has appeared that the expansion has begun to speed up again, as the repulsive effects of mysterious *dark energy* have come to dominate the expansion of the Universe. The Planck data also support the idea of dark energy acting against gravity. At present this accounts for about 70% of the entire mass of the Universe, and it will certainly increase in the future.

2.2.2 Gravitational Waves

It is worth mentioning that on the one-hundredth anniversary of the formulation of the theory of general relativity, as discussed in Subsection 2.1, we can further confirm its important implications. Figure 2.4 shows computer simulations of a possible generation mechanism of gravitational waves, which are actually distortions of spacetime in the vicinity of black holes. In fact, strong gravitational waves can arise during the merger of two massive black holes (of masses about 30 times larger than the mass of the Sun). Therefore, a large fraction of energy (∼5%, corresponding to three solar masses) has been released in this process in form of gravitational waves.

As seen in Figure 2.5, the measurements of experimental signals by two independent detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) in Hanford and Livingston (separated by 3000 km) are consistent with observations of a transient gravitational-wave strain[4] (with a peak of $\sim 10^{-21}$) in both time and frequency between 35 and 250 Hz (Abbot et al., 2016). For the first time this proves that the international experiment LIGO directly detected gravitational waves[5] originating several billions years ago from the merging of two black holes in the rotating binary system GW150914.

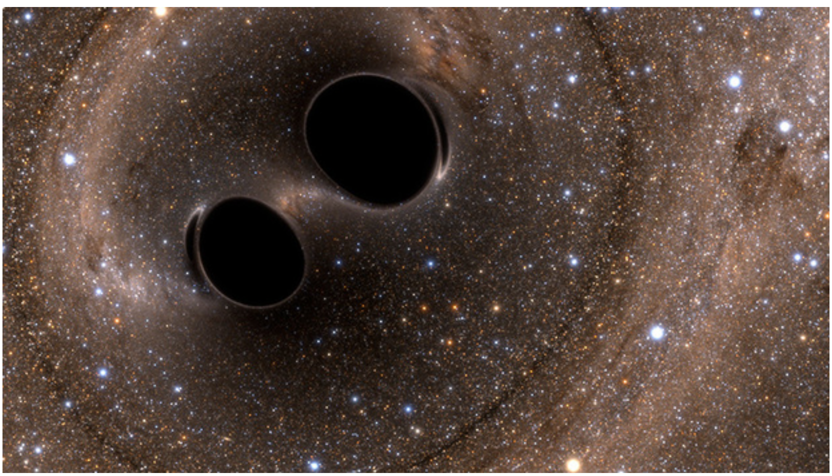

Figure 2.4 The generation of gravitational waves (LIGO).

4 For masses separated by a characteristic scale length L, the strain is a dimensionless parameter $\mathrm{d}L/L$, which is of the order of the amplitude of a gravity wave.

5 In 2017 the Nobel Prize in Physics was awarded to the American experimental and theoretical physicists Rainer Weiss, Kip Thorne, and Barry Barish for their role in the detection of gravitational waves.

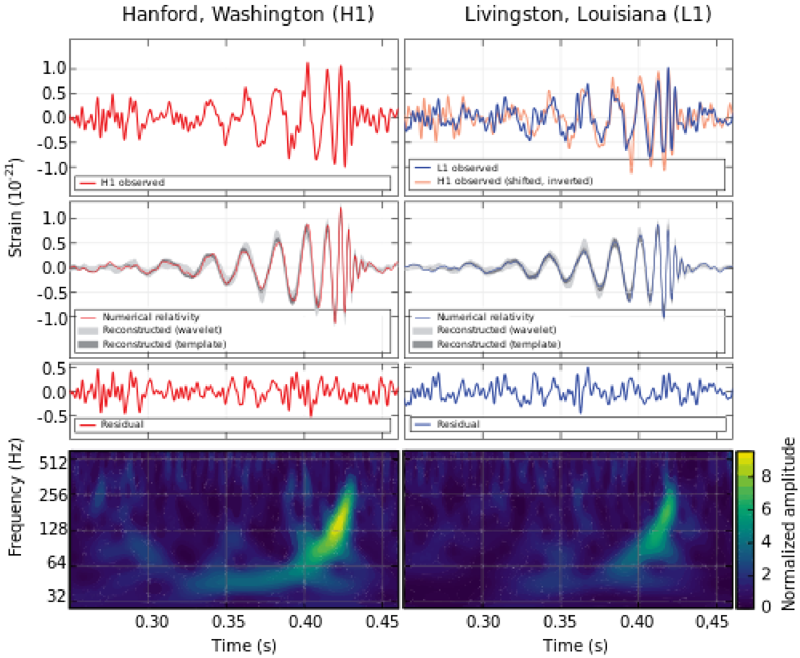

Figure 2.5 The discovery of gravitational waves, credit: LIGO.

2.2.3 The Question of the Origin of the Universe

Even though scientists are continuously searching for the ultimate explanation of the origin of the Universe, we do not understand the origin of spacetime and matter[6]. In particular, it seems that science is not yet prepared to answer at least three mysterious transitions. Namely:

- How was the primordial world created from nothingness?
- How did nonliving matter become living matter?
- How did living matter evolve into human beings?

One can even say that these three transitions seem to go far beyond any scientific issue, and even seem to be forbidden by contemporary science. Nevertheless, scientists are attempting to address them with bold new ideas. First, it seems that the physical laws of nature are essential to allow the birth and explain the childhood of the Universe. Therefore, if there was NOTHING

[6] Visit the Universe Forum, NASA, the Harvard Smithsonian Center for Astrophysics.

before the creation of the Universe, then the basic question is: how did the physical laws arise?

Secondly, how did these laws allow matter to evolve toward living matter? The origin of life still remains a mysterious transition from the thermodynamic point of view, even though observation of the *emergence* of complex structures would certainly help to resolve this issue. Finally, the third mysterious transition requires explaining how living matter produced conditions allowing the appearance of human beings with the ability to think about the origin of the world, even in terms of causality. Therefore, philosophers have tabled the concept of the causal Universe. For example, Ellis et al. (2013) have argued that

> The nature of causation is a core issue for science, which can be regarded as the move from a demon-centered world to a world based on reliable cause and effect, tested by experimental verification.

2.3 The Importance of the Unification of Physics for Cosmology

2.3.1 In Search of Quantum Gravity

In the natural sciences, a theory is usually a mathematical model that allows us to make predictions about the behavior of the world. For example, Einstein's theory of *gravity* accurately describes how matter responds to gravity on very large cosmological scales, i.e. up to distances of the entire Universe, estimated to be about 10^{27} m (roughly 10^{80} baryons, mainly nucleons: protons and neutrons). On the other hand, when going toward very small scales, we can well apply *quantum* theory, which perfectly describes both the tiny sub-atomic world ($\lesssim 10^{-10}$ m) and the nuclear microworld ($\sim 10^{-15}$ m). Actually, quantum theory makes very accurate predictions about the behavior of matter on small scales of distances in the range of about 42 orders of magnitude.

However, understanding the origin of the Universe requires developing a theory of spacetime that is related to matter on much smaller scales. In addition, these two theories are not complete and are not able to make predictions about the very earliest moments when the Universe was both extremely dense and extremely tiny, i.e., on scales of about 10^{-35} m that are about 20 orders of magnitude smaller than the size of the nuclear realm.

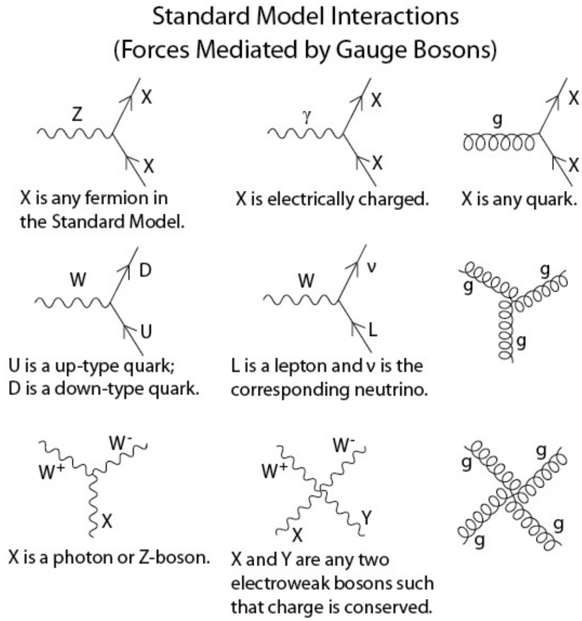

Figure 2.6 Feynman diagrams in the Standard Model.

2.3.2 The Standard Model of Forces

In the Standard Model of elementary interactions[7] we have three generations of elementary particles: three various types of quarks and leptons, together with gauge bosons, which are virtually mediating forces, as discussed in Appendix A. Namely, these gauge bosons responsible for the interaction of the respective forces (gluons g and photons γ for quarks, with heavy bosons Z and $W^{\pm}$ for leptons) are in the fourth generation, and finally the Higgs boson is in the fifth, providing the mass to particles, see Figure A.1.

We can have four basic types of interactions between particles: gravitational, strong, and weak and electromagnetic (electroweak) forces. These interactions are composed of elementary acts of two interacting point-like particles with another (third) gauge particle, as schematically shown in what we call Feynman diagrams. Figure 2.6 shows some examples of Feynman diagrams involving interactions of up-type or down-type quarks and gluons, fermions (e.g., nucleons: protons or neutrons), electrically charged particles with photons γ,

[7] From https://en.wikipedia.org/wiki/Standard_Model.

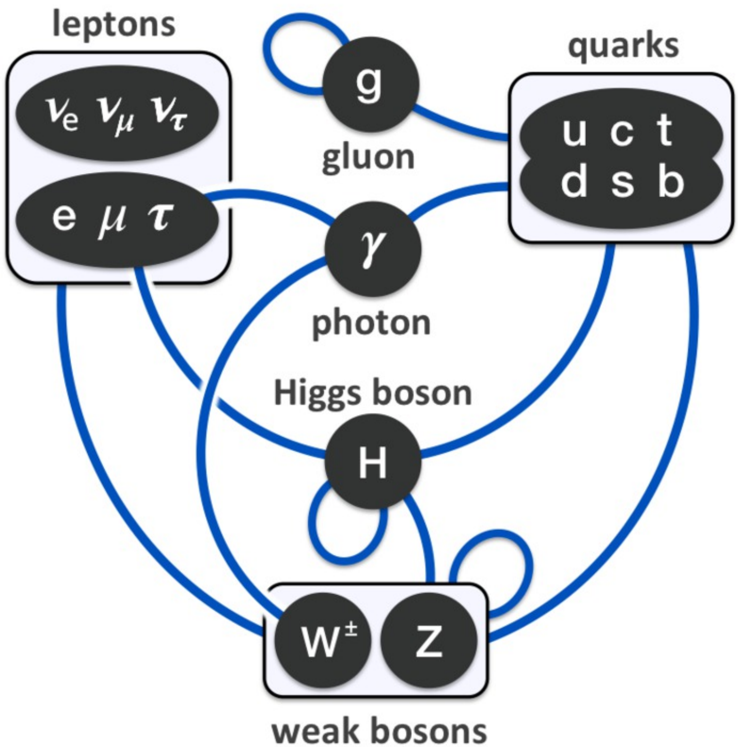

Figure 2.7 A summary of interactions between particles.

and leptons (e.g., electrons) with the corresponding neutrinos ν. All the interactions between elementary particles described by the Standard Model are summarized in Figure 2.7.

2.3.3 The Birth and Evolution of the Universe

The role of the elementary interactions during the evolution of the Universe[8] is depicted in Figure 2.8. One can see that the splitting of one force after the Big Bang into the four kinds of forces that we know today, after 1.38×10^{10} years of the evolution, happened in a very tiny fraction of the first second[9].

[8] From http://web.williams.edu/Astronomy/Course-Pages/330/images/forces.jpg
[9] Note that this period is even much smaller than the first three minutes of its evolution (cf. Hawking, 1988).

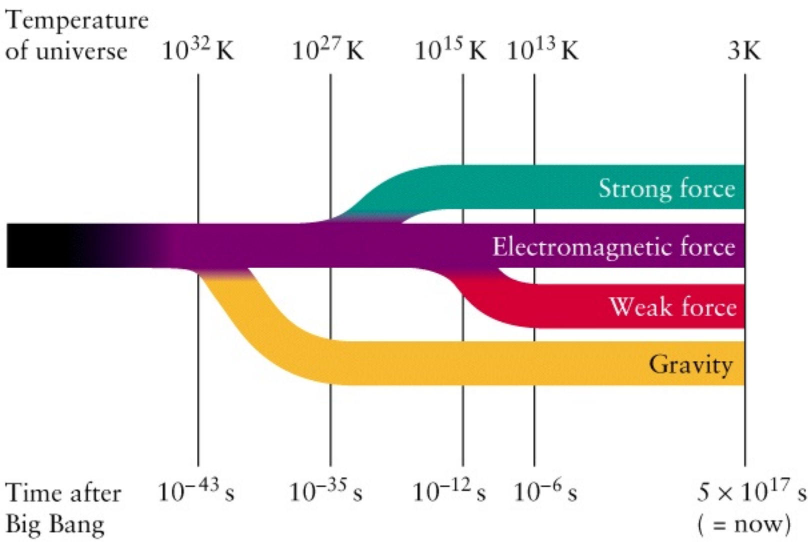

Figure 2.8 The birth and evolution of the Universe.

Strong forces should be limited only to the scales (nucleon size of ~10^{-15} m) in the microworld, while general relativity models long-range gravitational interactions on very large scales of up to the size (~10^{27} m) of the observed Universe[10].

Because the Universe has already expanded to that extremely huge size, gravitational forces (basically about 40 orders of magnitude weaker than strong nuclear forces) dominate the evolution of the Universe at present. However, at early stages of its evolution both forces resulted from an unknown simple law and could have been of a similar strength. The other long-range electromagnetic interactions between charged particles have already been unified with the short-range weak interactions responsible for the decay of nuclei (electroweak forces). Of course, the Grand Unification Theory (GUT) describing the unknown primordial force responsible for the creation of the Universe at a Planck scale of 10^{-43} s will facilitate a better understanding of the physical processes at very early stages of the history of our world.

[10] It is interesting that timescales are from 10^{-24} s in atomic nuclei to nearly 10^{18} s of the experimentally confirmed age of the Universe. This means a range of 42 orders of magnitude is the same as for spacescales; the masses span the range of about 83 orders of magnitude, between 10^{-30} kg for the electron mass and about 10^{53} kg for the of mass of the whole world (~10^{80} baryons, mainly nucleons: protons and neutrons with mass of ~10^{-27} kg); this range is roughly twice as large as the time or space scale range.

Admittedly, a very simplified cosmological model can already be calculated using the classical law of universal gravitation. Namely, in the Newtonian cosmological model a gravitational attractive force should be balanced by the repulsive force of the isotropic and homogeneous evolving Universe of mass M and a finite size R. Hence, the velocity of expansion, $\dot{R}(t) \equiv \mathrm{d}R/\mathrm{d}t$, can be obtained from the energy equilibrium equation $1/2\ \dot{R}^2 = GM/R$, where G is a gravitational constant. Therefore, assuming $R(0) = 0$ at an initial state $t = 0$, we obtain the size of the Universe growing in time according to the formula $R(t) = (9GM/2)^{1/3}t^{2/3} \propto t^{2/3}$. This means that the density of matter in the spherically expanding Universe shrinks in time evolution according to $\rho = (6\pi G)^{-1}t^{-2} \propto t^{-2}$; this also means that the density should increase infinitely at the initial singularity, when $R \to 0$, $\rho \to \infty$. However, some much more complex quantum models should also be considered for the primordial world.

2.4 Models for the Creation of the Universe

Scientists have always had difficulties with a singularity that could appear before the first known epoch of the evolution of our own Universe, as mentioned already in subsection 2.2.1. On the other hand, the mathematical structures used by physicists for modelling the microworld do not have any natural perceptible counterparts. For example, elementary particles (e.g., protons, electrons, and neutrinos or photons) are described by the wave functions ψ that are in turn elements of an abstract Hilbert space. What is observed results from a mathematical procedure of the projection of these abstract elements onto certain sub-spaces. It would be difficult to find a better metaphor than Plato's metaphor of shadows seen by prisoners on the wall of the cave (e.g., Macek, 2000).

The Schrödinger equation of the form

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi, \tag{2.1}$$

where $\hbar = h/(2\pi)$ and h is the fundamental Planck's constant; $\hat{H}$ is the Hamiltonian operator, $\hat{H}\psi = E\psi$ (involving kinetic T and potential energy U, the total energy $E = T + U$), describes the dynamics of the evolution of the wave function $\psi(x, y, z, t)$ corresponding to a particle with momentum $p = h/\lambda$ and energy $E = h\nu$, where λ is the (de Broglie) wavelength and ν is wave frequency in its configuration space located at position (x, y, z) and at time t; here i the imaginary basic unit, $\mathrm{i}^2 = -1$. According to a statistical interpretation, the square of the modulus of the (complex) wave function, i.e., $|\psi|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z$, is the probability of finding a particle in its volume element $d^3\mathbf{v} = dx dy dz$; the Hilbert space is normalized in the entire space of volume $\mathbf{V}$ by $\int |\psi|^2 d^3\mathbf{v} = 1$. One should note that the history and the amplitude of state for each particle could be described by the appropriate path integrals, corresponding to the concept of the classical action (Feynman and Hibbs, 1965).

2.4.1 Quantum Models for the Creation of the Universe

Now, using the three available universal physical constants — namely the gravitational constant G, the speed of light c, and the Planck constant $\hbar$, we can construct a quantity called a Planck length $l_P = \sqrt{G\hbar/c^3}$. Another quantity l_P/c is the respective Planck time scale, t_P. Because we do not have a quantum theory of gravitation a number of models for the creation of the Universe with the following characteristics have been proposed, including

- The quantum model (Hartle and Hawking, 1983)
  creation from 'nothing', ex nihilo
- Noncommutative geometry (Heller and Sasin, 1996)
  beginning is everywhere
- String theory, M-theory (Witten, 1995)
  collision of branes
- Cyclic (*ekpyrotic*) model (Steinhardt and Turok, 2002a,b)
  big bangs and crunches
- Eternal chaotic inflation (Linde, 1986)
  bubble of universes

Hartle and Hawking (1983) put forward the concept of the quantum wave function of the primordial Universe. They illustrated this point of view in a simple minisuperspace model with an invariant scalar field as the only gravitational degree of freedom. The authors of this model focus on the ground state with minimum excitation of an initial Universe on extremely small scales. Providing that the time is changed to imaginary values it, spacetime with a four-dimensional geometry becomes positive-defined. This allows us to obtain the path integral of the respective Euclidean action. In this way, Hartle and Hawking obtained finite nonzero probabilities of propagating from the ground (vacuum) state to the spectrum of possible excited states.

It is worth noting that below the Planck threshold $l_P = 1.6 \times 10^{-35}$ m $\sim 10^{-35}$ m and $t_P = 5.4 \times 10^{-44}$ s $\sim 10^{-43}$ s, in space and time, respectively, any time could be formally eliminated in the quantum model. Hartle and Hawking (1983) interpreted this scenario as the Universe without any boundary conditions. Moreover, because one can obtain the excited state from the vacuum state, they argue for the creation out of *nothing*, even *ex nihilo*. However, one should bear in mind that a quantum vacuum state is not actually 'nothingness' — indeed it could be interpreted as a 'sea' of various possibilities (Heller, 2009).

An alternative interesting solution for the origin of spacetime on extremely small scales has been proposed by Heller and Sasin (1996), who suggested that these critical values would correspond to a phase transition from a smooth commutative geometry to a rather singular noncommutative régime, with no space points and no time instances. Hence, noncommutative algebra is the other quantum gravity counterpart of the observable in the standard quantum theory, which can help in the application of quantization methods to the origin of the primordial Universe. Therefore, as Heller paradoxically put it: *the beginning is everywhere*.

Following the M theory that will be discussed in subsection 2.4.2, in the context of an initial Universe resulting from a *collision of branes*, Steinhardt and Turok (2002a,b) have proposed another interesting non-standard cosmological scenario. According to their proposed model, the Universe undergoes a sequence of cosmic epochs each of which begins with a created world with a standard big bang event, followed by a slowly accelerating expansion with radiation and matter domination periods, but ends by contraction with a crunch. This model is called *ekpyrotic*, because in ancient Greece's Stoic philosophy *ecpirosi* means 'escape from fire'. This endless cycle of *big bangs and crunches* would avoid any particular singularity, but is able to explain the approximate homogeneity of distribution of mass, instead of a hypothetical inflation following the Planck epoch. It is worth noting that the model produces the recently observed flatness of spacetime geometry, providing the energy needed to restore the Universe from the same vacuum state in the next cycle. These authors also assure us that, owing to acceleration, this continuously repeating cyclic solution is an attractor (Steinhardt and Turok, 2002b).

On the other hand, Linde (1986) suggests an eternally existing chaotic inflationary scenario, describing the Universe as a self-generating fractal that springs up from the multiverse. Namely, following Hartle and Hawking's wave function of the Universe, it can be shown that the large scale fluctuations of the quantum scalar field can generate an infinite process of self-reproducing primordial mini-universes. Therefore, Linde argues that there should exist an exponentially large number of causally disconnected mini-universes corresponding to all possible vacuum states followed by inflations. Because it seems improbable that only one such Universe is chosen in reality by compactification during the expansion, Linde argues that there exists a *bubble* of all possible *universes* that is always growing until a new Universe is created by chaotic inflation in the bubble.

Admittedly, in the last two models time is *eternal*, but it is difficult to verify these models according to the criterion of falsifiability required for any scientific theory (Popper, 1959).

2.4.2 String Theory

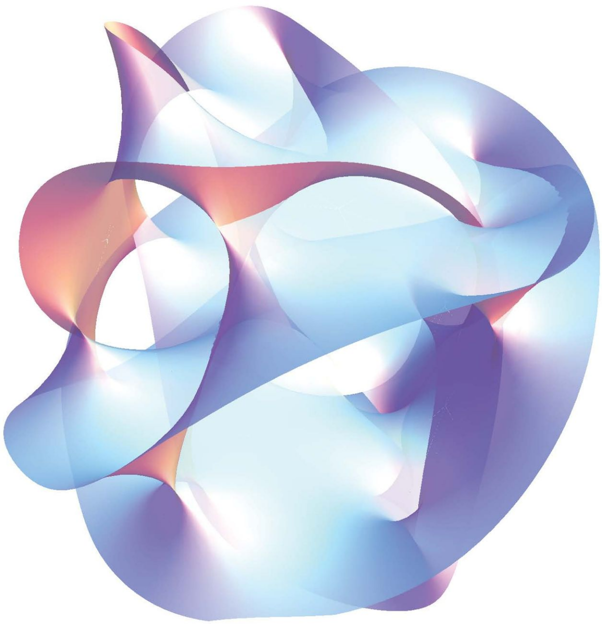

Figure 2.9 A projection of Calabi-Yau hyperspace.

To take into account gravitational forces on extremely small Planck scales (~10^{-35} m), where quantum effects should also play an important role, we can consider the new concept of multi-dimensional hyperspace. The additional dimensions are responsible for gravitation, but they are so small as compared with macroscopic scales that they are not discernible in the observed world. We here talk about curled-up or ‘compact’ dimensions. An example of such hyperspace is the manifold (shape) of Calabi-Yau (e.g., Yau, 1978), which is useful in ‘string theory’. The two-dimensional projection of such six-dimensional hyperspace is depicted in Figure 2.9.

It is therefore worth noting that some of the scholars in physics are working on a new string theory of space, time, and matter (e.g., Witten, 1995) that may help us formulate a better model of fundamental forces in nature and to understand the origin of the Universe. Admittedly, string theory is based on new ideas that have not yet been tested. The theory assumes, for example, that the basic particles in nature are not point particles, but are vibrating on structures shaped like *strings*. Consequently, interactions in the subatomic world are represented by the lines of point-like particles in the Standard Model or a world sheet swept up by closed strings in string theory, as schematically depicted in Figure 2.10.

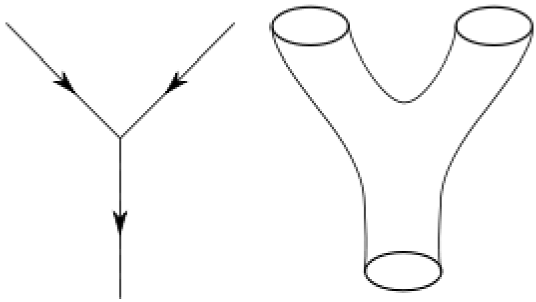

Figure 2.10 Interaction in the subatomic world: world lines of point-like particles in the Standard Model or a world sheet swept up by closed strings in string theory.

2.4.3 Predictions of M-theory

Superstring theory requires that space has *more* than the *three* dimensions in which we move. According to the version of the theory proposed by Witten (1995), called M-theory, the particles and forces (except gravitation) that are acting in the world are confined to *three* spatial dimensions and one time dimension, and the *extra* (six) compact dimensions are responsible for gravitational interactions.

This string theory (with 10 dimensions) has led to some bizarre new scenarios for the origin of the Universe. In one scenario, the Big Bang could have been triggered when our own Universe *collided* with a ‘parallel universe’ made of these *extra* dimensions. Scenarios like these are very speculative and require 11-dimensional hyperspace. Although super string theory is still in development and remains untested, it stimulates astronomers to look for new forms of evidence.

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3

Nonlinear Dynamics and Fractals

3.1 Nonlinear Dynamics

The concept of a dynamical system stems from classical mechanics, where the equations of motions are obtained from the known general laws, e.g., from the classical ones of Newton or the relativistic equations of motions, assuming chosen values of initial conditions. Today, general dynamical system theory is an important subject of interdisciplinary science, and a dynamical system is any defined part of the world that is evolving in time. In general, a dynamical system consists mathematically of an abstract state space or phase space and a dynamical rule that specifies the immediate future trend of all state variables, given the present values of the state variables. The system is deterministic if there is a unique consequent to every state, otherwise the system could be random or stochastic, if there are more possibilities chosen from some probability distributions.

There are two basic types of deterministic dynamical systems — namely, with the evolution of the state of the system in continuous and discrete time independent variables. In a general mathematical framework, a continuous dynamical system (called a flow) is described by the differential equations

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}), \tag{3.1}$$

where the vector field $\mathbf{F}(\mathbf{x})$ is the function of a vector state $\mathbf{x}$ in the N-dimensional phase space, $\mathbf{x} = (x_1, \ldots, x_N)$. The solution of the ordinary differential equations is trajectory, $\mathbf{x}(\mathbf{t}) = [x_1(t), \ldots, x_N(t)]$, which involves derivative on time t only (in contrast to the partial differential equations, where both time, vector states, and possibly its derivatives are independent variables). Here the overdotes denote ordinary differentiation with time $\dot{\mathbf{x}} \equiv \mathrm{d}\mathbf{x}/\mathrm{d}t$. In particular, a steady state with a zero vector field is called a fixed point, $\mathbf{F}(\mathbf{x}^*) = \mathbf{0}$. Naturally, we require that the partial derivatives $\partial F_i/\partial x_j$, for $i, j = 1, \ldots, N$ are smooth enough for the existence of a unique solution (Lipshitz's condition) for a given initial state $\mathbf{x}(0)$. For nonautonomous systems a time dependence of the vector field can be taken into account by adding the extra dimension to the system, taking formally $x_{N+1} = t$. For example, an oscillating system (e.g., pendulum) is two-dimensional, $N = 2$, because two variable are necessary for its description (position x and velocity $v \equiv \dot{x}$), but a new variable, time t, is needed for the forced oscillating system, which makes it often much more complex, $N = 3$.

On the other hand, a discrete system is defined by an iterated difference equation, with N-dimensional function $\mathbf{f}$ called a map,

$$\mathbf{x}_{n+1} = \mathbf{f}(\mathbf{x}_n), \tag{3.2}$$

with an integer-valued discrete time variable $n = 0, 1, 2, \ldots$. In particular, any k-periodic solution satisfies the following condition: $\mathbf{x}_{n+k} = \mathbf{x}_n$ (for $k = 1$, we have a fixed point $\mathbf{f}(\mathbf{x}^*)$). Maps are convenient tools for analyzing flows described by differential equations that are necessarily invertible. Because, iterated functions in maps can also be non-reversible, they can exhibit surprisingly complex behavior even for a one-dimensional case, N=1. The well-known example is the logistic map, as illustrated in point 3.1.1, which shows complex solutions resulting from bifurcations, depending on a control parameter of the system. Because for a continuous system any chaotic behavior is excluded for $N < 3$, an important illustration of nonlinear three-dimensional Lorenz system, which exhibits deterministic chaos, bifurcations, and intermittent behavior is provided in point 3.2.1.

Finally it is worth noting that only in a very special case of a linear system does the function $\mathbf{F}(\mathbf{x})$ in Equation (3.1), or $\mathbf{f}(\mathbf{x})$ in Equation (3.2) depend only on the first power of the independent variable $\mathbf{x}$. A nonlinear system is merely a negation of a linear system[1]. Linearity is often defined using the superposition principle. Namely, in the case of a linear system, if we have two solutions, we automatically have another solution that is a combination of these two solutions[2]. Therefore, the fundamental tools of linear analysis (such as a Fourier decomposition) are no longer available for the nonlinear systems; each solution must be coped with as a whole. In particular, any solution multiplied by a constant number is also realized in the linear system. Philosophically, this means that the effect is proportional to the acting cause. But in nonlinear systems the results may be more than linear or

[1] Stanisław Ulam, the Polish mathematician, ironically said that calling a science 'nonlinear' sounds like calling zoology 'the study of non-human animals'.

[2] Steven H. Strogatz has noted that according to the superposition principle, when we listen to two favorite songs at the same time, our pleasure is doubled, which is certainly not always true.

less than linear[3]. Consequently, a small cause can result in dramatic events (popularized as the 'butterfly effect'), and on the contrary, a bigger cause can hopefully produce tiny effects. Anyway, basically the principle of causality needs rethinking in nonlinear dynamical systems.

3.1.1 A Logistic Map

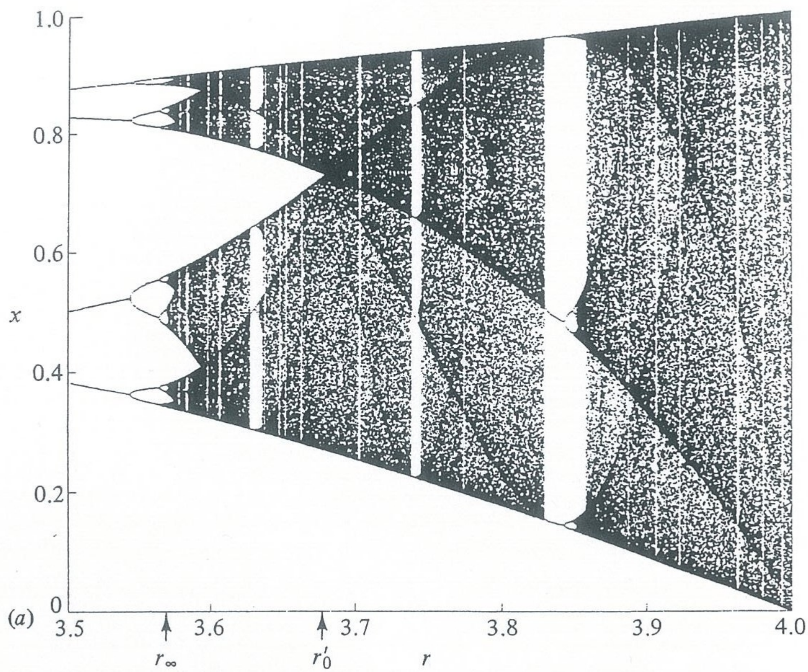

Figure 3.1 Bifurcation for the logistic map.

Maps defined in Equation (3.2) are often very simple models of complex phenomena in many systems in natural, economic, and even social sciences. For example, the logistic map

$$x_{n+1} = f(x_n) \tag{3.3}$$

with an elementary (quadratic) function $f(x) = rx(1 - x)$ has been proposed

[3] For example, as the populations of consumers limit or enhance the resources on the market.

by Robert M. May, arguing that *intuition may be enriched by seeing the wild things that simple nonlinear equations can do* (May, 1976). One can note that for the control parameter $0 \leq r \leq 4.0$, Equation (3.3) maps the interval $0 \leq x_n \leq 1.0$ onto itself, for any iteration step $n = 0, 1, \ldots, \infty$. If the variable x_n is small the next population x_{n+1} will be larger, if however x_n is large its growth will substantially be reduced.

Naturally the evolution of the system, with a nonlinear function, depends on the values of the control parameter r. Clearly, for small $r \leq 1$ the population $x_n \to 0$ as $n \to \infty$, for $1 < r < 3$ its value is stable and grows settling to a steady state (a fixed point, $x^* = 1 - 1/r$), which looses its stability at $r = 3$, where two new stable fixed points arise $x^* = (r + 1 \pm \sqrt{(r-3)(r+1)})/(2r)$. When two (or even more) solutions appear for a certain control parameter of the system, we say that we have bifurcation. It can be verified analytically that these two fixed points are stable for $r = 1 + \sqrt{6}$.

Figure 3.1 depicts the bifurcation diagram (calculated numerically) with stable solutions of the variable $0 \leq x \leq 1.0$ for the logistic map with some larger values of the control parameter r above 3.5 (e.g., Ott, 1993, Fig. 2.11 (a)). This behavior clearly exhibits a period doubling route to *chaos*. Period doubling starts at $r = 3.0$ (period 2^2 is born at $r = 1 + \sqrt{6} \approx 3.449$) and successive bifurcations (for periods 2^k) come faster and faster, converging to $r_\infty \approx 3.57$, where long-term behavior becomes *nonperiodic* (period 2^∞). But suddenly, for $r > r_\infty$, at a critical value $r_c = 1 + 2\sqrt{2} \approx 3.8284$ period 3 (periodic window) is born (*tangent* bifurcation, type I **intermittent** chaos for $r \lesssim r_c$) (e.g., Strogatz, 1994, Fig. 10.4.4). Hence both *chaos* and *order* (nonperiodic and periodic solutions) are intertwined.

3.2 Deterministic Chaos

CHAOS ($\chi\acute{\alpha}o\varsigma$) according to Strogatz (1994) is (see the excellent popular book by Stewart (1990):

- NON-PERIODIC long-term behavior
- in a DETERMINISTIC system
- that exhibits SENSITIVITY TO INITIAL CONDITIONS.

More precisely, we say that a bounded solution $\mathbf{x}(t)$ of a given dynamical system, $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ of Equation (3.1), is SENSITIVE TO INITIAL CONDITIONS if there is a finite fixed distance $d > 0$ such that for any neighborhood $||\Delta\mathbf{x}(0)|| < \delta$, where $\delta > 0$, there exist (at least some) other solutions $\mathbf{x}(t) + \Delta\mathbf{x}(t)$ for which for some time $t \geq 0$ we have $||\Delta\mathbf{x}(t)|| \geq d$.

This means that there is a fixed distance d such that, no matter how precisely one specifies an initial state, there exists a solution of a dynamical system starting from a nearby state (at least one) that gets a distance d away.

Given $\mathbf{x(t)} = \{x_1(t), \ldots, x_N(t)\}$ from Equation (3.1), any positive finite value of Lyapunov exponents (or equivalently metric entropy)

$$\lambda_k = \lim_{t\to\infty} \frac{1}{t} \ln\left|\frac{\Delta x_k(t)}{\Delta x_k(0)}\right|, \tag{3.4}$$

where $k = 1, \ldots N$, implies chaos.

3.2.1 The Lorenz Model

One example comes from the dynamics of irregular flow in viscous fluids, which is still not sufficiently well understood. It appears that the behavior of such systems can be rather complex: from equilibrium or regular (periodic) motion, through intermittency (where irregular and regular motions are intertwined) to nonperiodic behavior. Two types of such nonperiodic flows are possible, namely chaotic and hyperchaotic motions. As discovered by Lorenz (1963) deterministic chaos exhibits sensitivity to initial conditions leading to the unpredictability of the long-term behavior of the system (the 'butterfly effect'). Please note that in the seminal paper by Lorenz (1963) with the abstract shown in Figure 3.2 the term *chaos* did not yet appear, rather *nonperiodic* behavior was used instead.

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

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(Manuscript received 18 November 1962, in revised form 7 January 1963)

ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

Figure 3.2 Abstract of Lorenz's (1963) seminal work.

Starting from complex basic partial differential (Navier-Stokes, heat conduction, and continuity) equations, by employing some reasonable approximations, Lorenz (1963) obtained the following three simple but nonlinear ordinary differential equations:

$$\left.\begin{array}{lcl} \dot{X} & = & \sigma(Y - X) \\ \dot{Y} & = & -XZ + rX - Y \\ \dot{Z} & = & XY - bZ. \end{array}\right\} \qquad (3.5)$$

In this simplified system, $X(t)$ denotes a time amplitude of the potential of the velocity of a viscous horizontal fluid layer in the vertical gravitational field heated from below, with the normalized (dimensionless) Rayleigh number r, proportional to an initial temperature gradient δT_0, which is a control parameter of the system. Similarly, $Y(t)$ and $Z(t)$ correspond to the two lowest-order amplitudes of the deviation from the linear temperature profile of the layer (of height h) during the convection. The other parameter $\sigma = \nu/\kappa$ is the ratio of the kinematic viscosity ν to thermal conductivity κ (the Prandtl number) characterizing the fluid and $b = 4/(1+a^2)$ is a geometric factor related to the aspect ratio a of the convected cells. Admittedly, Lorenz (1963) only took three of several coefficients appearing in the lowest-order of the bispectral Fourier expansion (cf. Saltzman, 1962).

Because the light coming from the Sun after impinging on the Earth's surface certainly heats the atmosphere, Lorenz expected to examine the feasibility of very-long-range weather prediction. But even with this crude model the obtained numerical solutions were extremely complicated. For some values of the model parameters, $r = 28$, $\sigma = 10$, $b = 8/3$, the solution is *nonperiodic*, as illustrated in Figure 3.3. This means that the system exhibits sensitivity to initial conditions (resulting from the lack of Lyapunov stability). It is worth noting the importance of nonlinear coupling terms XZ and XY in Equations (3.5), which are responsible for advection (when the fluid moves its properties are changed). In this example, one can verify that complex behavior does not necessarily result from complicated laws of nature, but could often follow from a simple but nonlinear law. Moreover, for the first time, we see here an asymptotic solution in a form of a *strange attractor* that has fractal structure presented in Figure 3.4.

3.2.2 Hyperchaos

Hyperchaos is a more complex nonperiodic flow, which was discovered by Macek and Strumik (2014) in the generalized Lorenz system previously proposed (Macek and Strumik, 2010). Mathematical and physical aspects of

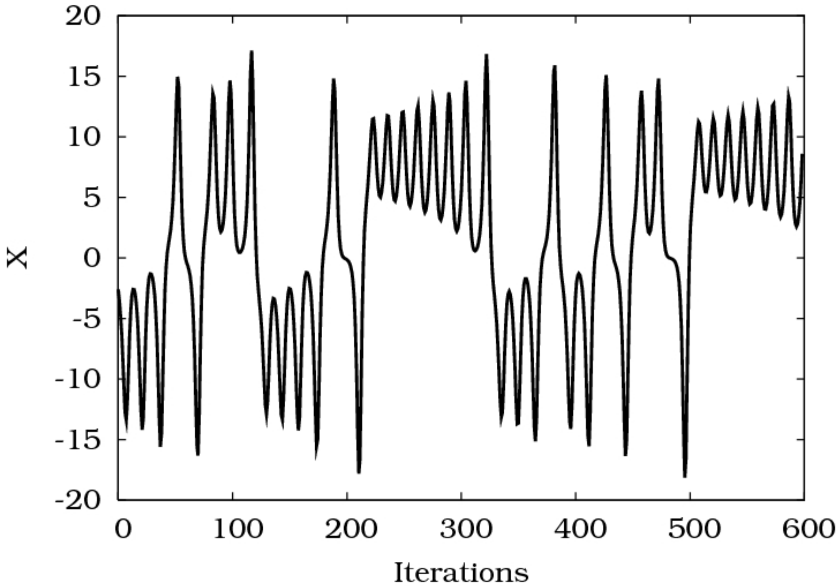

Figure 3.3 Times series for *X* in the Lorenz system.

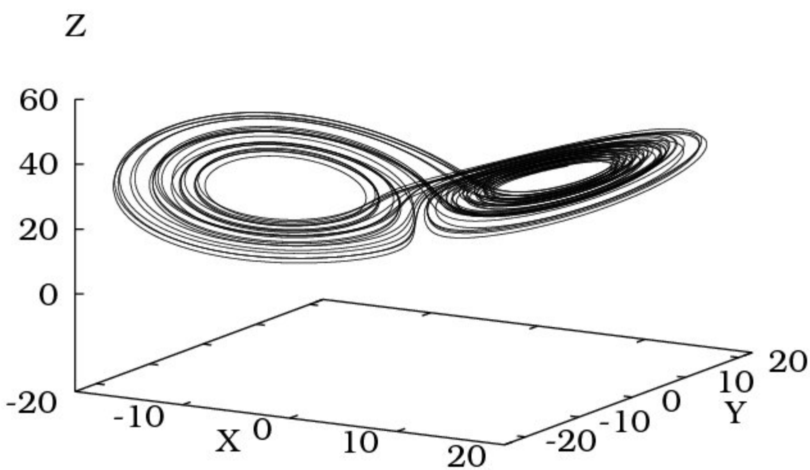

Figure 3.4 The strange Lorenz attractor.

this new low-dimensional model of hydromagnetic convection together with the detailed derivation from the basic partial differential equations, including the magnetic diffusion equations and naturally the anisotropic tension of the magnetic field lines, has been recently addressed in detail by Macek (2018).

Within the theory of dynamical systems transitions from fixed points to periodic or nonperiodic flows often occur in a given system through bifurcations, intermittency, resulting in a turbulent irregular behavior of the nonlinear system. In fact, we have identified type I and III intermittency (Pomeau and Manneville, 1980) in the generalized Lorenz model of hydromagnetic convection, as also discussed in the previous papers (see Macek and Strumik, 2010, 2014; Macek, 2015). It would be interesting to look for the remaining basic type II intermittency and the respective Hopf bifurcation in this model.

The following ordinary differential equations are obtained by Macek and Strumik (2010):

$$\left.\begin{aligned} \dot{X} &= -\sigma X + \sigma Y - \omega_0 W \\ \dot{Y} &= -XZ + rX - Y \\ \dot{Z} &= XY - bZ \\ \dot{W} &= \omega_0 X - \sigma_{\mathrm{m}} W. \end{aligned}\right\} \tag{3.6}$$

In addition to the standard Lorenz (1963) system given by Equations (3.5), a new time dependent variable W in Equations (3.6) describes the profile of the magnetic field induced in the convected magnetized fluid. We have also introduced the second control parameter proportional to an initial horizontal magnetic field strength B_0 applied to the system, more precisely defined here as a basic dimensionless magnetic frequency $\omega_0 = \upsilon_{A0}/\upsilon_0$, which is the ratio of the Alfvén velocity $\upsilon_{A0} = B_0/(\mu_0\rho)^{1/2}$, with a constant magnetic permeability μ_0 and mass density ρ, to a characteristic speed $\upsilon_0 = 4\pi\kappa/(abh)$. Naturally, besides $\sigma = \nu/\kappa$, the magnetized viscous fluid is characterized by an analogue parameter $\sigma_{\mathrm{m}} = \eta/\kappa$, defined as the ratio of the magnetic resistivity η to the thermal conductivity κ (related to the magnetic Prandtl number, $Pr_{\mathrm{m}} = \sigma/\sigma_{\mathrm{m}}$).

The results of the more recent paper illustrate how all these complex motions can be studied by analyzing this simple model (Macek and Strumik, 2014, Fig. 1). For example, for a chosen value of $\sigma_{\mathrm{m}} = 3$ (other parameters have the same values as for the classical Lorenz model, $\sigma = 10$, $b = 8/3$), Figure 3.5 plots the largest Lyapunov exponent, calculated according to Equation (3.4), depending on the control parameters ω_0 and r. Convergence of the asymptotic solutions of Equations (3.6) to equilibria described by fixed points ($\lambda_1 < 0$) is shown in black, to periodic (limit cycles) solutions ($\lambda_1 = 0$) – in violet/blue color (see the color bar for $\lambda_1 = 0$), to chaotic (nonperiodic) solutions ($\lambda_1 > 0$) – in a color, consistently with the color bar scale, from violet/blue to yellow.

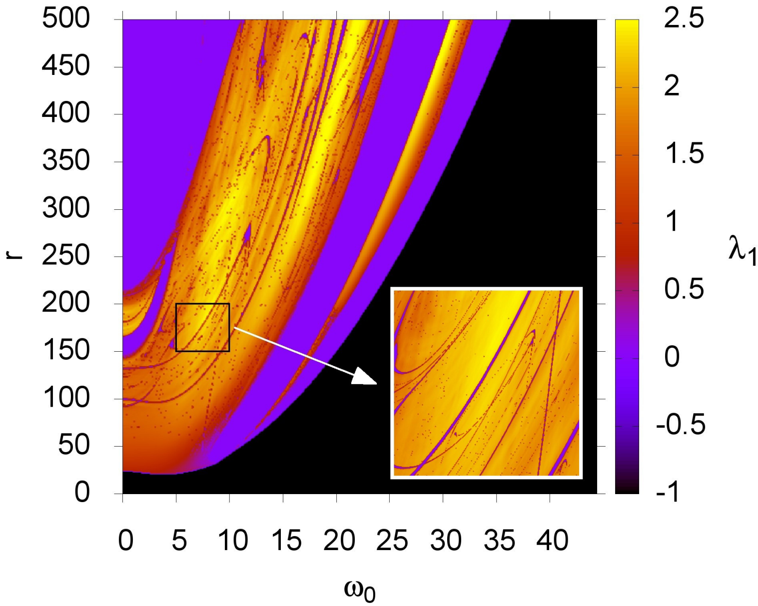

Figure 3.5 Color-coded dependence of the long-term asymptotic solutions of the generalized Lorenz system on the control parameters ω_0 and r parameters (for $\sigma_m = 3$). Equilibria (fixed points) (with a negative largest Lyapunov exponent, $\lambda_1 < 0$) are shown in black, periodic solutions ($\lambda_1 = 0$) – in violet/blue, and (nonperiodic) chaotic solutions ($\lambda_1 > 0$) – in a color, on the color bar scale, from violet to yellow. Fine structures are shown in the inset, as taken from (Macek and Strumik, 2014).

For the panel an enlargement of the region bounded by black lines is shown in the right-bottom part of plots. Fine structures are shown in the inset. This proves that various kinds of complex behavior are closely neighbored in the space of control parameters ω_0 and r.

Convection appears naturally in plasmas, where electrically charged particles interact with the magnetic field. Therefore, the obtained results could be important for explaining dynamical processes in solar sunspots, planetary and stellar fluid interiors, and possibly for plasmas in nuclear fusion devices. Generally speaking, nonlinear differential equations or iterated discrete maps are useful models of some phenomena appearing naturally in the contexts in biology (e.g., animal population), economics, including finance theory (e.g., Peters, 1996), and social sciences.

3.3 Fractals and Multifractals

Let us now move on to a basic concept of a *fractal* coined from the Latin adjective *fractus* and the corresponding verb *frangere*, which means 'to break into irregular fragments' (see Mandelbrot, 1982, p. 4); Mandelbrot always argued that fractal geometry is important for understanding the structure of nature describing, for example clouds, mountains, and coastlines (e.g., Mandelbrot, 1982, p. 1).

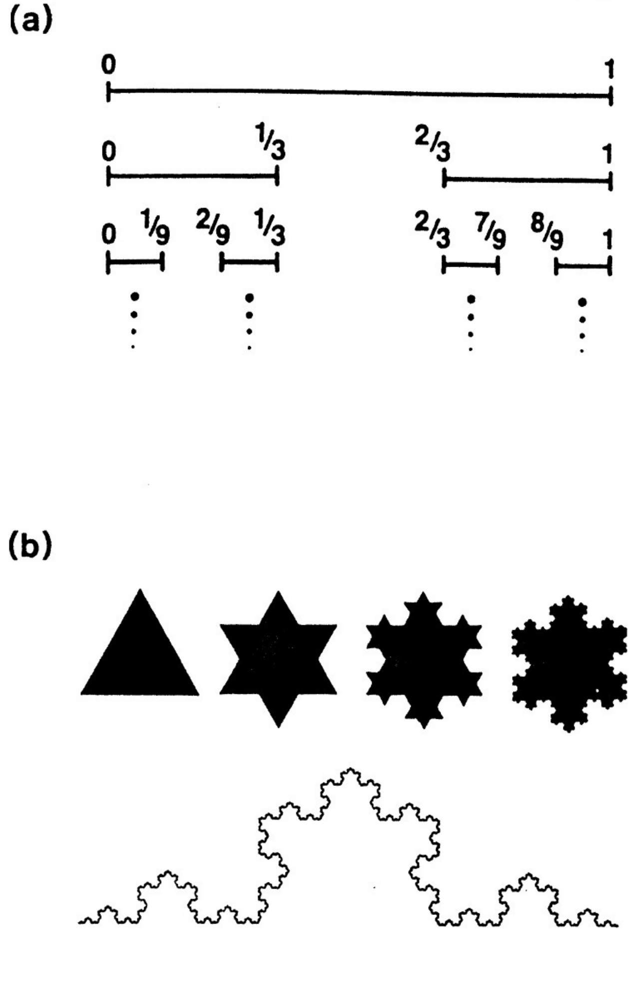

Figure 3.6 Self-similar fractals of the Cantor (a) and Koch (b) sets.

Fractal structure is obtained recursively using a simple rule. The initial stages of the construction of two typical fractals in one- and two-dimensional space are schematically illustrated in Figure 3.6 for a middle Cantor (a) and a Koch triangle (b) sets, respectively, which are also discussed in many textbooks (e.g., Strogatz, 1994; Ott, 1993). First, as proposed by the German mathematician Georg Cantor in 1883, let us take a unit closed interval on a one-dimensional line and remove its open middle third, but necessarily leaving the endpoints behind. Second, we remove the open middle thirds of both closed smaller intervals, and in each of the following k-th step this produces 2^k closed (more and more narrower) intervals of length $(2/3)^k$, where $k = 1, \ldots, n$. Now imagine that the repetitions never end, one obtains the limiting set that consists of the intersection of all such closed intervals. Provided that $n \rightarrow \infty$, the resulting set has structure at arbitrarily small scales; the remaining elements during the construction are separated by various gaps. Surprisingly enough, two paradoxically opposite topological properties of the Cantor set (called also a dust) can be reconciled: the set itself is totally disconnected (without any closed intervals), but arbitrarily close to each elements one can always find another neighboring element (there are no isolated points).

Further, it is worth noting that each element of this set is specified by its location at successive steps, in the left (denoted by zero) or right (marked by one) fragment. One now sees that elements of the Cantor set are equivalent to various infinite sequences of zeros and ones, and can be put into one-to-one correspondence with the elements of the entire initial interval (in binary representation). Because common sense has some difficulty in comparing countable with uncountable infinity, this is somewhat strange that the Cantor set is uncountable, notwithstanding its total length equal to zero (the length of all the removed parts is equal one). Mainly because of this paradox, such sets are commonly called strange fractals, even though one can also construct fractals with length or in general volume (strictly a Lebesgue measure) different than zero[4].

Figure 3.6 (b) shows another interesting snowflake curve obtained on a plane by adding onto sides of an initial equilateral triangle additional triangles that are three times smaller, after removing as before open middle thirds of any side. Blowing up this van Koch curve by a factor of three results in its length four times as large, and hence the length of perimeter of the triadic Koch island increases and becomes ultimately infinite, despite the fact that the area of course remains finite. Surprisingly, the arc length between any two elements

[4] Similar fractal sets with zero Lebesgue measures constructed starting from a triangle or a full square on a two-dimensional plane were proposed by the Polish mathematician Wacław Sierpiński (1882–1969) in 1916.

of such a Koch set is also infinite. Therefore, because every element of this set is located infinitely far from any other element, the length cannot be used to identify the elements of such a strange fractal. Anyway, the concept of dimension should be modified as compared with a standard topological dimension useful in the Euclidean linear geometry.

Mandelbrot noted that a fractal (Hausdorff) dimension[5], which plays a central roles in case of fractal sets, exceeds the topological dimension, D_{T}. However, a somewhat different definition of a fractal set is generally accepted. Namely, we can say that a **fractal** is a rough or fragmented geometrical object that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally *self-similar* and independent of scale, described by a fractal dimension.

Namely, to measure the size of any set of volume V we ask how many elements (cubes) of size l in phase space is needed to cover the set. Because the volume $V(l) \propto l^{D_{\mathrm{F}}}$, one expects that the number of cubes $N(l)$ should follow a power low dependence, $N(l) \propto l^{D_{\mathrm{F}}}$. Therefore, the fractal (capacity) dimension D_{F} is defined by

$$D_{\mathrm{F}} = \lim_{l\to 0} \frac{\ln N(l)}{\ln 1/l}, \tag{3.7}$$

and is calculated by taking the limit of the quotient of the logarithm change in object size and the logarithm in scale as the limiting scale approaches zero. For example, the fractal dimensions of the Cantor and the Koch sets are $D_{\mathrm{F}} = \ln 2/\ln 3 \approx 0.63\ (> 0)$ and $D_{\mathrm{F}} = \ln 4/\ln 3 \approx 1.26\ (> 1)$ i.e., greater than the respective topological dimensions, $D_{\mathrm{T}} = 0$ and 1. As is known, the later non-integer dimension describes sufficiently well the length of the rocky western coast of Great Britain as a function of diminishing scale size; in reality the lowest scale is admittedly limited (Mandelbrot, 1967).

In addition to a usual probability measure $p_i(l)$ associated with a given scale l, using a one parameter pseudoprobability measures

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^{N} p_i^q(l)}, \tag{3.8}$$

where q is any real number $-\infty < q < +\infty$, the generalized dimension is defined by

$$D_q = \frac{1}{q-1} \lim_{l\to 0} \frac{\ln \sum_{i=1}^{N} (p_i)^q}{\ln l}. \tag{3.9}$$

[5] Strictly speaking, the Hausdorff dimension is more involved that a usual fractal capacity dimension. The boxes needed to cover a set may vary in sizes and one needs to take a supremum of the cover of the set.

It is easy to verify that the capacity dimension of Equation (3.7) is recovered for $q = 0$, $D_F = D_0$. The higher order dimensions are related to dynamics of the system. For large positive q the cubes in the phase space are more strongly weighted in the generalized entropies in the numerator of Equation (3.9), while for a negative q the smaller fluctuations of the quantities responsible for $p_i(l)$ are amplified.

For example, if a measure p is applied to the left part of the interval and $1-p$ to the right remaining part in Figure 3.6 (a) the function $\tau(q) \equiv (q-1)D_q$ is equal to

$$\tau(q) = \frac{\ln[p^q + (1-p)^q]}{\ln 1/3} \tag{3.10}$$

and the generalized dimension of the middle-thirds weighted Cantor set is given analytically, with $D_0 = \ln 2/\ln 3$.

Now, provided that a probability measure $p_i(l)$ satisfies also a power low dependence, say on a singularity strength α_i, i.e., $p_i(l) \propto l^{\alpha_i}$, a number of necessary elements in a continuous range of α (i.e., between α and $\alpha + \mathrm{d}\alpha$) should depend on some function $f(\alpha)$, namely $N_l(\alpha) \sim l^{-f(\alpha)}$, we can define a multifractal spectrum of scaling indices by a double limit (Falconer, 1990)

$$f(\alpha) = \lim_{\varepsilon\to 0}\lim_{l\to 0}\frac{\ln[N_l(\alpha+\varepsilon) - N_l(\alpha-\varepsilon)]}{\ln 1/l} \tag{3.11}$$

Figure 3.7 (a) depicts the generalized dimensions D_q as a function of any real order q and (b) shows the multifractal spectrum $f(\alpha)$ versus the singularity strength α. These universal functions have the following properties: (1) the maximum value of $f(\alpha)$ is the capacity dimension D_0, (2) $f(D_1) = D_1$, the value corresponding to the information dimension, $D_1 = \lim_{l\to 0}[\sum_{i=1}^{N} p_i(l)\ln p_i(l)]/(\ln l)$, and (3) the line joining the origin to the point where $\alpha = D_1$ on the $f(\alpha)$ curve that is weighting various generalized dimensions is tangent to this function (see Ott, 1993, p. 308). We see that the **multifractal** is a set of intertwined fractals, and the self-similarity of multifractals is scale-dependent, with the spectrum measuring the relative weights of various multi-fractals, showing how the dimension varies across the strange set.

One can note that the multifractal spectrum $f(\alpha)$ is related to the derivative of the function $\tau(q) \equiv (q-1)D_q$ by the Legendre transformations $\alpha(q) = \tau'(q)$ and $f(\alpha(q)) = q\alpha(q) - \tau(q)$. For example, for the weighted Cantor set we have the analytical expression

$$\alpha(q) = \frac{1}{\ln 1/3}\,\frac{p^q\ln p + (1-p)^q\ln(1-p)}{p^q + (1-p)^q} \tag{3.12}$$

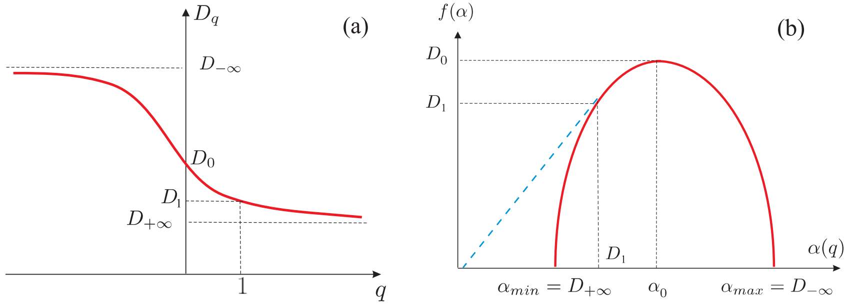

Figure 3.7 The universal functions of the generalized dimensions D_q and the multifractal spectrum $f(\alpha)$.

and using Equation (3.10) the multifractal spectrum $f(\alpha(q))$ is obtained analytically.

3.3.1 Multifractal Models for Turbulence

A deviation from a strict self-similarity is also called INTERMITTENCY, and that is why a generalized two-scale weighted Cantor set has been applied for modeling intermittent turbulence in fluids (Macek, 2007, 2012).

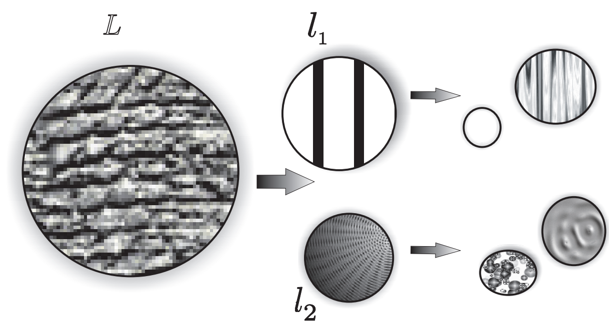

Figure 3.8 Schematics of binomial multiplicative processes of cascading eddies.

We consider a standard scenario of cascading turbulent eddies, as schematically shown in Figure 3.8 (cf. Meneveau and Sreenivasan, 1991). We see that a large eddy of size L is divided into two smaller *not necessarily equal* pieces of size l_1 and l_2. Both pieces may have different probability measures, p_1 and p_2, as indicated by the different shading. At the n-th stage we have 2^n various eddies. The processes continue until the Kolmogorov scale is reached (cf. Meneveau and Sreenivasan, 1991; Macek, 2007; Macek and Wawrzaszek, 2009). In particular, space filling turbulence could be recovered for $l_1 + l_2 = 1$. Ideally, in the inertial region of the system of size L, $\eta \ll l \ll L = 1$ (normalized), the energy is not allowed to be dissipated directly, assuming $p_1 + p_2 = 1$, until the Kolmogorov scale η is reached. However, in this range at each n-th step of the binomial multiplicative process, the flux of kinetic energy density ε transferred to smaller eddies (energy transfer rate) could be divided into nonequal fractions p_1 and p_2.

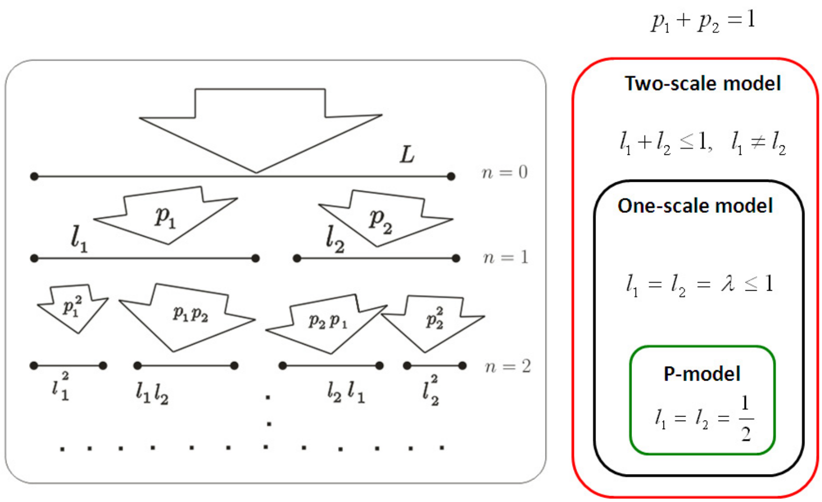

Figure 3.9 The generalized two-scale weighted Cantor set model for turbulence.

Naturally, this process can be described by the generalized weighted Cantor set, as illustrated in Figure 3.9 taken from (Macek, 2007). In the first step of the two-scale model construction, we have two eddies of sizes l_1 and l_2 satisfying $p_1/l_1 + p_2/l_2 = 1$. Therefore, the initial energy flux ε_0 is transferred to these eddies with the different proportions: $\varepsilon_0 p_1/l_1$ and $\varepsilon_0 p_2/l_2$. In the next step the kinetic or magnetic energy flux is divided between four eddies in the following way: $\varepsilon_0(p_1/l_1)^2$, $\varepsilon_0 p_1 p_2/(l_1\ l_2)$, $\varepsilon_0 p_2 p_1/(l_2 l_1)$, and $\varepsilon_0(p_2/l_2)^2$. At nth step we have $N = 2^n$ eddies and partition of energy ε can be described by the following binomial formula (e.g., Macek, 2012):

$$\varepsilon = \sum_{i=1}^{N} \varepsilon_i = \varepsilon_0 \sum_{k=0}^{n} \binom{n}{k} \left(\frac{p_1}{l_1}\right)^{(n-k)} \left(\frac{p_2}{l_2}\right)^{k}. \tag{3.13}$$

For any q in Equation (3.9), one obtains $D_q = \tau(q)/(q-1)$ by solving numerically the transcendental equation (e.g., Ott, 1993),

$$\frac{p_1^q}{l_1^{\tau(q)}} + \frac{p_2^q}{l_2^{\tau(q)}} = 1, \tag{3.14}$$

which is only somewhat more general than the analytical solution given by Equation (3.10). In particular, for the one-scale multifractal model with $l_1 = l_2 = \lambda$, we have $D_q = -\ln(p_1^q + p_2^q)/\ln\lambda$, and a special case for $\lambda = 1/2$ is called P-model, as classified on the right side of Figure 3.9. Obviously, only for equal scales together with equal weights ($p_1 = p_2 = 1/2$) there is no multifractality, and we have a monofractal with a fractal dimension given by Equation (3.7).

3.3.2 Models and Observations

Naturally, to compare the solution of Equation (3.14) with experimental data, one needs to estimate the probability p_i appearing in Equations (3.8) and (3.9) for any given dynamical system and fit the theoretical spectrum $f(\alpha)$ to the obtained experimental spectra defined by Equation (3.11).

For a fluid with velocity u the transfer rate of the energy flux $\varepsilon(l)$ at a given scale l is usually estimated by (Frisch, 1995)

$$\varepsilon(l) \sim \frac{|u(x+l) - u(x)|^3}{l}. \tag{3.15}$$

Therefore, to each ith eddy of size l in turbulence cascade ($i = 1, \ldots, N = 2^n$) we associate a probability measure defined by

$$p_i(l) = \frac{\varepsilon_i(l)}{\sum_{i=1}^{N} \varepsilon_i(l)}. \tag{3.16}$$

Using Taylor's hypothesis, one can argue that $p_i(l)$ can be regarded as a probability that at a position $x = Vt$, at time t, where V is the average fluid speed, a given magnetic flux is transferred to a spatial scale $l = V\Delta t$.

The generalized probability measures $p_i(l)$ depending on scale l can be constructed using magnetic field strength fluctuations in the following way (Burlaga, 1995). Namely, normalizing a time series of magnetic field data $B(t_j)$,

$$p_i(l) = \frac{1}{N} \sum_{j=1+(i-1)\Delta t}^{i\Delta t} B(t_j), \tag{3.17}$$

for $i = 2^{n-k}$ ($k = 0, 1, \ldots, n$) and $j = 1, \ldots, N = 2^n$ is calculated with the successive average values $\langle B(t_j, \Delta t)\rangle$ of $B(t_j)$ between t_j and $t_j + \Delta t$, for each $\Delta t = 2^k$ (e.g., Macek et al., 2011, 2014). Again this quantity can roughly be interpreted as a probability that the magnetic flux is transferred to an eddy of size $l = V\Delta t$.

Burlaga (1995) has shown that in the inertial range the average value of the qth moment of B at various scales l should scale as

$$\langle B^q(l) \rangle \sim l^{\gamma(q)}, \tag{3.18}$$

where the exponent γ is related to the generalized dimension D_q, $\gamma(q) = (q-1)(D_q-1)$. Following this method, using these slopes for each real q, the values of D_q can be determined using Equation (3.18). Alternatively, as explained in Subsection 3.3, the multifractal function $f(\alpha)$ versus scaling index α, Equation (3.11), which exhibits universality of the multifractal scaling behavior, can be obtained using the Legendre transformation. It is worth noting, however, that we obtain this multifractal universal function directly from the respective quotients of the logarithm of pseudoprobability of Equation (3.8) and the logarithm of the scale.

This direct method was extensively used in various situations in solar wind magnetized plasmas based on space missions penetrating various regions of the solar system (see, Macek, 2012; Macek and Wawrzaszek, 2009; Macek et al., 2011, 2012). In this way, based on a wealth of data acquired from Helios in the inner heliosphere and especially from deep space Voyager 1 and 2 spacecraft in the outer heliosphere, we have shown that turbulence is intermittent in the entire heliospheric system, even at the heliospheric boundaries (Macek et al., 2014). However, it appears that the heliosphere is immersed in a relatively quiet local interstellar medium. Therefore, after crossing the heliopause (on 25 August 2012), which is the ultimate boundary separating the heliospheric and interstellar plasmas, Voyager 1 only detected smoothly varying magnetic fields. As expected this change in the behavior of plasma parameters (with a frozen-in magnetic field) was confirmed by the crossing of the heliopause by Voyager 2 in 2018.

3.4 Implications for Cosmology and the Creation of the Universe

The theory of nonlinear dynamics and chaos has also been applied in astronomy and astrophysics, including gravitationaly bound (many body) planetary systems, irregular nonlinear stellar pulsations and accreting systems, and stellar and galactic clusters dynamics (e.g., Regev, 2006). Paradoxically, one should bear in mind that the Euclidean three-dimensional space filled-up with a constant density of mass distribution would have produced the infinite Newtonian gravitational forces. Admittedly, despite the discovery of large massive inhomogeneous structures with large spatial empty voids, which are common features of astrophysical observations, the standard cosmological model based on the theory of general relativity also employs a somewhat similar approximation claiming that the Universe is homogeneous, at least on some very large scales. On the other hand, the available data satisfy power low distributions of mass with various exponents that are substantially lower than three, ranging from a value greater than 1 to about 2; the latter very special value of quasi-fractal dimension D_F given by Equation (3.7) is supported by luminous radiation data and is consistent with a flat Universe in thermodynamic equilibrium; in addition, this certainly satisfies the Copernican principle. Various monofractal distributions of galaxies have been reported in the astrophysical literature (e.g., Maddox, 1987), but it seems that the clustering structures with number $N(l)$ at distance l are better explained by the multifractal spectrum of dimensions $f(\alpha)$ defined by Equation (3.11) with $N(l) \propto l^{-f(\alpha)}$ (e.g., Jones et al., 2005). It also seems that the universal multifractal function for galaxies is basically similar to that identified by NASA's Voyager missions in the Solar System's plasmas (see Macek et al., 2014).

Moreover, in contrast to Newtonian theory, in general relativity one deals with pseudo-Riemannian spaces, and one has to modify the concepts described for classical nonlinear dynamical systems. In particular, the definition of the Lyapunov exponents should be reformulated (e.g., Szydlowski and Heller, 1994) to consider the stability of cosmological systems with Big Bang scenarios, including inflationary solutions, or for example, geodesic flows in Wheeler-De-Witt superspace.

Based on scientific experience, I have argued that a simple but possibly **nonlinear law** (cf. Macek, 2000), within the theory of *chaos* and (multi-)fractals, can describe a hidden ORDER for the creation of the COSMOS, at the Planck epoch, when space (at a scale of 10^{-35} m) and time (10^{-43} s) originated (Macek, 2013, 2016).

To summarize, based on space, astrophysical, and even cosmological applications, one can say that

- **Nonlinear** systems exhibit complex phenomena, including bifurcation, intermittency, and CHAOS.
- Fractals can describe complex shapes in the real word.
- Strange *chaotic* attractors have fractal structure and are sensitive to initial conditions.
- Within the complex dynamics of the fluctuating intermittent parameters of turbulent media there is a detectable, hidden ORDER described by a generalized Cantor set that exhibits a multifractal structure.

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PART TWO

RELIGION

4
Science and Religion

Needless to say, there are essential differences between science and religion. While the natural sciences try to explain the world in terms of laws by using a language of mathematical structures avoiding supernatural causes, religion aims to express the Divine Reality by using human language. Even though the *methods* of science and religion are *different*, studies on quantum reality suggest that the two can mutually complement each other and help us approach the Truth (d'Espagnat, 1983). The methods of science are usually the subject of the philosophy of science, which is also concerned with the foundations, justification, aims, and implications of science.

4.1 The Philosophy of Science

The philosophy of science is a branch of philosophy devoted to the presuppositions, foundation, methods, purpose, and essential features of scientific knowledge. There is no single view among philosophers on what the central problems of the philosophy of science are, including the ontological and epistemological issues of the relation of science to truth, and even whether science can really reveal the truth at all. Therefore, to consider the relation between science and religion a suitable philosophy of science will be a good adviser.

Perhaps besides a positivistic approach, in my view, most scientists seem to believe in the ability of science to grasp reality and recognize that the aim of science is to approach the Truth in the classical sense of Aristotle, *adaequatio rei et intellectus*. However, the *a priori* limitation in reasoning raised by Kurt Gödel (1906–1972) with his two incompleteness theorems, should weaken our beliefs in the proofs of any axiomatic scientific system. Alternatively, the validity of general scientific theories can be tested only from a series of experimental verifications, but the implications *a posteriori* are never quite certain.

4.1.1 Antiquity

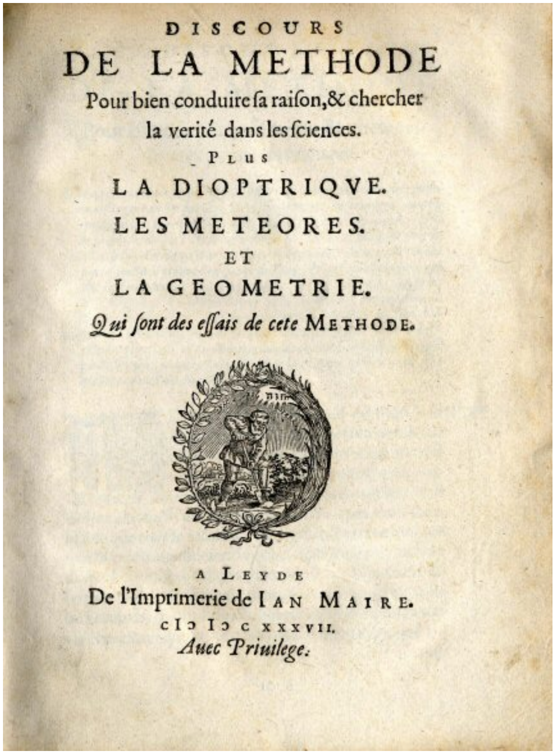
As we know, science originated with early Greek philosophy. However, Socrates (469–399 BC), the teacher of Plato, primarily concerned himself with the basic moral concept of virtue, and it seems that he did not like to think about the laws of nature. Socrates invented the elenctic method (refutation based on cross-examining) and the maieutic method (to stimulate critical thinking), which played an important and very useful role in approaching ultimate Truth (see the dialogue *Theaetetus* written by Plato circa 369 BC).

Archimedes of Syracuse (288–212 BC), in turn, pointed to the experimental verification of the ideas proposed by Plato (see p. 4). Therefore, one may say that the natural sciences originated from the Platonic-Archimedean tradition. On the other hand, Aristotle provided the foundations of logic, but his notions of matter and form are simply ignored in modern science; besides the efficient cause, the material, formal, and final causes are not considered anymore in the natural sciences. In addition, the concept of *aether* was falsified in the twentieth century.

4.1.2 The Modern Era

In the seventeen century, René Descartes (1596–1650), the father of rationalism, starting from methodical doubts found a proper method for human thinking. His 'Discourse on the Method (of Rightly Conducting One's Reason and of Seeking Truth in the Sciences)', written 1619–1620 (see Figure 4.1), underlined the central role of reason as opposed to the senses and established a framework for scientific knowledge (Descartes, 1637). This seminal work can be considered the onset of modernity, which was soon characterized by the foundation of the mathematical-natural sciences by Isaac Newton (1643–1727), Johannes Kepler (1571–1670), and Galileo Galilei (1564–1642).

It seems that the foundation of classical physics was possible only owing to mathematical studies on the ideal notion of infinity. In fact, the invention of differential and integral calculus with infinitesimally small changes of quantities (e.g., velocity and acceleration) allowed the study of the motion of celestial objects. Admittedly, the idealized motions of material points have their counterparts in sensible experience, but it appeared that observations acquired via the senses can often be misleading. In particular, it appears that the motion of bodies does not necessarily require an acting force. Newton claimed



DISCOURS
DE LA METHODE
Pour bien conduire ſa raiſon, & chercher
la verité dans les ſciences.
PLUS
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui ſont des eſſais de cete METHODE.

A LEYDE
De l'Imprimerie de IAN MAIRE.
cIↃ IↃ c XXXVII.
Auec Priuilege.

Figure 4.1 Cartesian *Discourse on Method* (1637).

that every material point persists in its state of uniform motion in a straight line that is basically equivalent to rest, until the action of other bodies compels it to change that state (the first law of motion).

Contrary to Aristotle's conviction that every motion requires a cause, according to Newton's second law force is only necessary to change that state and is proportional to acceleration. The coefficient is called mass and is rather a measure of the inertia of the material body, but this certainly has a somewhat different meaning that the concept of matter in Aristotelian philosophy. Even though it later turned out that, strictly speaking, mass is not a constant of motion (for very large velocities), Newtonian physics can be considered an important scientific paradigm for all material bodies moving at velocities small compared to the speed of light, where relativistic physics should be taken into account.

4.1.3 Contemporary Times

Cartesian seventeen-century science was based on reason as a subject of scientific activity. In more modern times a transition could be observed from subjective to somewhat more objective knowledge, or more precisely, to knowledge that is intersubjectively sensible. This important revolution was performed only in the last century by Karl R. Popper (1902–1994) with his epistemology developed within the approach of critical rationalism. As is well known, his main philosophy of knowledge is adequately described by the parable of three worlds (Popper, 1978). Besides the first world of physical objects, including the things-in-themselves (*noumena* in Kantian philosophy), and the second world of mental objects and events related to individual subjective knowledge (in Cartesian philosophy), there may exist a third world of abstract meta-objects (another form of being), which is at least partially autonomous from the first and second worlds.

Namely, this third realm consists of all valuable information acquired by humankind, including scientific works, tools, models, and theories, as well as of artistic works, and other products of thought; this world is hence related to intersubjective knowledge, paradoxically, as he says 'to epistemology without a knowing subject', which grows with critical selection (Popper, 1972). Popper also suggested that the existence of this third realm is only possible in an open society based on knowledge, science, and understanding (Popper, 1945). Karl Popper also argued that the central question in the philosophy of science is *falsifiability*. This means that, at least in principle, every scientific theory should be able to be proven false.

According to Thomas Kuhn (1922–1996), who did not limit science to hypotheses, such falsification is often performed by a paradigm shift (Kuhn, 1970). The Copernican revolution in astronomy was a good example of the development of science. It seems that the twentieth century's scientific revolution provides new important paradigms in the history of science, not only with special and general relativities, quantum theory, but even in classical physics.

In accordance with Ockham's razor, scientists strove for the simplest available explanations. Based on modern studies on nonlinear dynamics and elementary particles, as described in the previous two chapters of this book, scientists are convinced that all complex phenomena in nature stem from ONE simple principle. The multiplicity of various dynamical behaviors is hence an illusion. Physicists are continuously searching for such a fundamental rule from which other particular natural laws can be derived. This is illustrated by the modern concept of nonlinear dynamics and deterministic chaos in

Chapter 3, in the case of the logistic map and the Lorenz system described in subsections 3.1.1 and 3.2.1, respectively. There is also general agreement that laws should result from symmetries. Therefore, in the modern physical sciences the aesthetic criterion is in a way taken into consideration in search of Truth, even though experimental verification of theories can never be neglected. This fundamental rule should be compact and simple, thereby resulting in a wealth of structural details, as for example in the case of fractals and multifractals considered in Section 3.3. Therefore, we also believe that the Universe most probably originated from this unique law at the Planck time, as proposed in Section 1.1.3.

Quantum Reality

The question of how to interpret quantum mechanics is not yet understood clearly in terms of reality[1] (d'Espagnat, 1983).

Contrary to the founders of quantum mechanics[2], especially Niels Bohr (1885–1962) (Bohr, 1935), Albert Einstein (1879–1955) with Boris Podolski and Nathan Rosen defended a simplified version of realism in nature. They postulated the concept of hidden parameters and formulated the well-known EPR paradox[3], depicted in Figure 4.2 for two well-separated quantum objects that cannot communicate faster than the speed of light (Einstein et al., 1935).

In a nutshell, their reasoning can be summarized as follows. Because no influence of any kind can propagate faster than the speed of light, and assuming that induction is a valid way of reasoning in quantum mechanics, we cannot reconcile two obvious premises. One is realism: phenomena are caused by a physical reality whose existence is independent of human observers, and the second is local causality (assuming the independence of well-separated objects). One cannot reject any of these self-evident truths. Hence one is led to conclude that the description of reality as given by a wave function is not complete (Einstein et al., 1935).

As discussed in Section 2.4, any quantum object can be described as a particle or a wave, which can lead to a paradoxical observation. Erwin Schrödinger (1887–1961) proposed the famous thought experiment, illustrated using the idea of a hypothetical cat suspended between life and death, which is known as 'Schrödinger's cat' in the academic literature. We can also

[1] https://wonderverse.home.blog/2019/10/06/quantum-reality-to-be-and-not-to-be/

[2] Ironically, Einstein belongs among the many scientists who have provided essential contributions to the foundations of quantum mechanics; he was awarded the Nobel Prize in 1921 for his discovery of the law of the photoelectric effect.

[3] Taken from Mark Garlicks/Photo Library, Getty Images

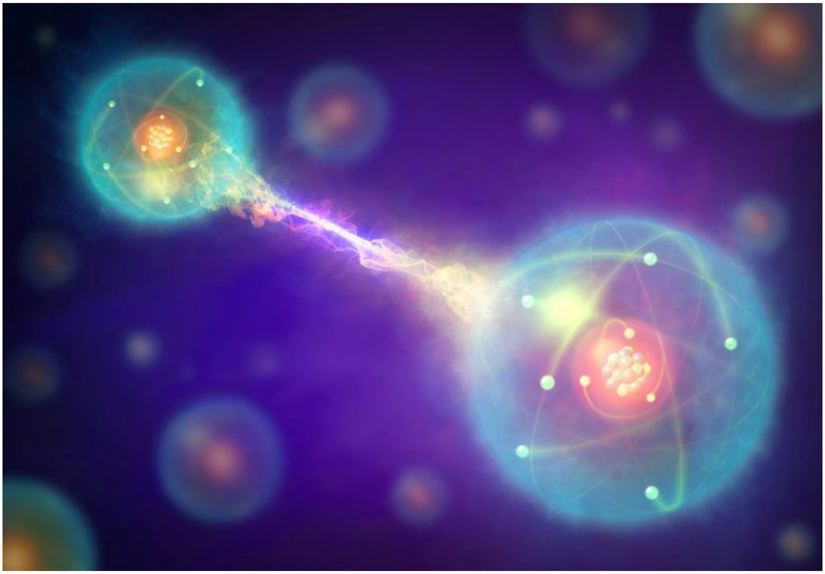


Figure 4.2 The EPR paradox in physics.

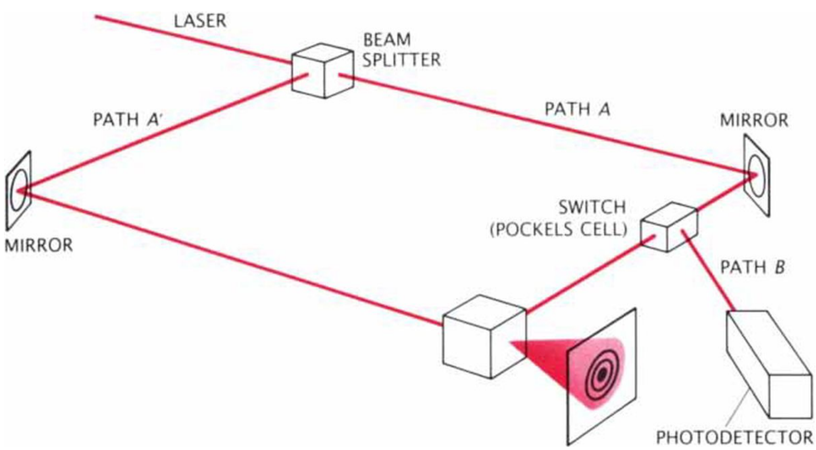


Figure 4.3 A delayed-choice experiment on quantum reality.

think about the photon, i.e., the fundamental unit of light that can behave like either a particle or a wave, as depicted in Figure 4.3 (Shimony, 1988). According to quantum mechanics the photon can exist in an ambiguous state until a measurement is made. If a particle's property is measured, the photon behaves like a particle, and if a wavelike property is measured, the photon behaves like a wave. In our experiment the photon impinges on a beam splitter and on half-silvered mirrors. The photon has a probability of, say, one-half of passing through the mirror and a probability of one-half of being reflected. If the photon is reflected and interferes with itself (paths A and A'), we observe its wavelike property (e.g., an interference pattern) and nothing dramatic happens. On the contrary, if the photon passes through the mirror (path A) and is deflected into path B, it is detected by the photodetector in a box. In this case, the detection actuates a device that breaks a bottle of cyanide, which in turn kills the cat in the box. Remarkably, due to a certain switch, the photon could not have been informed whether to behave like a particle (if the switch is on) or like a wave (if the switch is off) until it is registered. Hence, it cannot be determined whether the cat is dead or alive until the box is opened. Similarly, one cannot determine the spin until the particle is measured and, surprisingly enough, the measured value depends on the observer.

Many laboratory tests show that, despite Einstein's objections, the strange character of the quantum world must be accepted. Referring to the notion of scientific falsifiability of Popper and Kuhn's revolution by changes of paradigms, I wish to note that can we learn from mistakes in physics: *Errare humanum scientificum est*. Paradoxically, only the greatest scientist as Einstein could make an error so important and useful in the question of the reality in the microworld. Admittedly, our superficial observations in reliance on the senses, when applied to the natural sciences, especially without any mathematical background, can often be misleading.

Maybe an anecdote from the history of science in the twentieth century will be useful here. Niels Bohr, who served as the director of a special university institute in Copenhagen, organized seminars on quantum theory that later resulted in the Copenhagen interpretation of wave function, as given in Equation (2.1) and discussed in Section 2.4. However, during the presentations of visiting scientists, he was often well behind the other participants in understanding the reasoning of a given speaker. Even when the other participants helped him to follow the content of the lecture, he still was not able to understand. Finally, Bohr concluded that he began to understand. However, his understanding was quite different from the speaker, and at the end it turned out that it was Bohr who was right, contrary to the erroneous explanations proposed by the audience.

Anyway, it seems to me that quantum theory requires a new philosophical concept of the *existence* of the elementary constituents of the microworld. Namely, a quantum object can be in an entangled state described by the superposition of the wave functions ψ of Equation (2.1) in the form

$$\psi = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle \tag{4.1}$$

that is a composition of two opposite properties, e.g., spin up (relative to an axis) with the probability α^2 and spin dawn with the probability β^2, satisfying $\alpha^2 + \beta^2 = 1$, this is respectively symbolized by the parable of the cat suspended between life and death.

In my view, this scientific paradigm requires a **new** philosophical concept of BEING based on modern metaphysics as I proposed (2000). Certainly, this novel metaphysics must go beyond the classical ontological principles of identity, excluded middle, and noncontradiction. It seems that physics can be of help in understanding this difficult issue in search for a new ontology based on modern science, because none of these basic principles is satisfied in the quantum world. If we have a quantum state describing particles with combinations of opposite properties, this situation can be realized in nature. In fact, in the quantum world there are some mysterious phenomena that are well described by a pair of entangled states. Two opposite properties of particles are realized with some probabilities. It is strange, but there is no contradiction in having both states simultaneously. Referring to the celebrated quote from Hamlet: *to be or not to be*, the basic question for the new metaphysics is: how *to Be and Not-to-Be*?

4.2 The Philosophy of Religion

Since Aristotle's philosophy, metaphysics has been related to the philosophy of religion, even though the term came into general use only in the nineteenth century, after philosophy and religion were distinguished from theology. Today, the philosophy of religion is a distinct subject of philosophy devoted to a critical examination of the nature of religion, including the belief in God and its implications. Naturally, there are many philosophical positions with regard to the main questions of religious beliefs ranging from theistic, through agnostic, to atheistic approaches.

Because the methods of science and religion are different, some philosophers would like to separate religion from science. Admittedly, the natural sciences try to explain the world in terms of laws by using a language of mathematical structures avoiding supernatural causes. On the other hand religion aims to express the Divine Reality by using natural human language. In my view, both science and religion tend to approach Truth and hence science can provide an important contribution to the study of religion.

Because science as a whole can be considered a *locus theologicus*, Heller (1996) in particular has proposed an integrated view of science, philosophy, and theology and has even constructed a program toward such an INTERDISCIPLINARY field. The philosophy and resulting theology of Heller is certainly a proclamation of *rationalism*. Focusing on God and the created Universe, he continuously underlines that not only science but also faith should be rational and argues that theology and science both have a common objective: understanding humanity and the world created by God. In this way, Heller has even put forward a proposal for a new theological discipline: a THEOLOGY OF SCIENCE, which should look at the sciences from the theological point of view and from the moral perspective (Macek, 2010, 2011).

Therefore, a new science and theology can help us understand the following problems related to religion:

- nonlinear causality

  In view of nonlinear dynamics and chaos, as discussed in Chapter 3, the effect can be out of proportion of the cause and, in particular, the theological understanding of God as the First Cause needs rethinking, see page 5. In addition, for high-dimensional nonlinear dynamical systems deterministic and random forces result in a similar irregular behavior; chance and probability can hardly be distinguished.
- chance and probability

  Therefore, the concept of God, provided by the argument of an intelligent (design) project, who avoids any accidental actions would be taken with caution; in the Mind of God chance and cause should be indistinguishable. One can say that when *God plays dice* the mathematical laws of nature are assigned to the world (cf. Stewart, 1990).
- evolutionary creation

  Further, in view of the Universe in Modern Science, as discussed in Chapter 2, any static world view or belief must be reconciled with the evolution of the world; there is no contradiction between evolution and creation; evolutionary creationism seems to be appropriate view.
- existence of the world

  Even though the existence of the natural world is different from supernatural reality, quantum theory gives an example that matter and ideas cannot be separated.
- reality

  In the view of modern science searching for reality in the quantum microworld, considered in Chapter 4, page 59, one cannot expect that classical metaphysics is suitable for a description of reality on very small scales (Macek, 2000).

Finally, in my own experience, the contemporary natural mathematical sciences are continuously renewing our thoughts about God and the meaning of human life (Macek, 2009, 2010). In reality, the well-known Latin aphorism attributed to Gottriedd Wilhelm Leibniz (see pages 10 and 15) expresses a general truth that *when God counts the world comes to be*:

Cum Deus calculat fit mundus.

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5

Questions of Meaning

The question of meaning arises in the life of every human being and is certainly related to science and religion, as discussed in Section 4. In a nutshell, while science provides knowledge, religion is to give meaning to our lives. However, the human is immersed in the Universe[1], as depicted in Figure 5.1. Therefore, the meaning of life and the meaning of the Universe are genetically related (Heller, 2010).

[1] See the contribution to the blog *Wonderverse* of our interdisciplinary scientific group https://wonderverse.home.blog/2018/12/23/wonderful-universe/

Figure 5.1 The human in space.

5.1 The Universe and Meaning

From the philosophical point of view, not only does every human require some justification for his existence — so does the Universe itself. Therefore, we ought to look for the meaning of the world in the mystery of rationality; the meaning given by God to every existing being is the *justification* of the existence of the Universe. This is why we can experience that really

something **does** *exist* instead of nothing (cf. G. W. Leibniz).

As we have pointed out in Chapter 2 (see Subsection 2.3.3 and Figure 2.8), in the beginning said 'something' (the primordial Universe) was extremely small. Now the entire Universe is huge, but its origin still remains a Big Mystery. One may even say that from the scientific point of view, the birth of this primordial world from nothingness is a forbidden transition (page 21) — notwithstanding which, the Universe does really exist.

For many scientists the question of the meaning of the world should go far beyond any scientific issue. But from the point of view of philosophy and theology, the problem of the origin of our world is the key problem of the human existence. Therefore, one can say that the whole Universe is the unique Word of God that gives *meaning* to humanity, history, and the world (Heller, 2010). The Reason that was in the beginning penetrates every being; existence results from the rationality of Divine Thought. Naturally, Heller is convinced that faith should not be in separation from science. For him science as a whole is a *locus theologicus*.

5.1.1 The Universe and Humankind

Following Heller (2010) some important observations are relevant to the problem of meaning:

- We should note a certain relation of the Universe to *thought*. Surprisingly enough, although human thinking is limited to a relatively very short time, it now enables us to recover the whole of cosmic history, which began nearly 14 billion years ago. Moreover, human values can be realized in the context of the Universe, which is an incarnation of sensible thought.
- We are deeply immersed in the Universe: life appeared during the evolution of the Solar System (3.8 billion years ago), later came the first *brain* awareness event, i.e., when the first human was born.
- Moreover, human individuals are able to act following their own *will* and thoughts, including feelings that are very characteristic for our life. Therefore, any choice of *meaning* is a demand of *rationality*, because the

rejection of the meaning should be deemed a betrayal of human reason. In this way, when asking about meaning, we are also asking about God, who is continuously providing meaning to the whole Universe.

- In scientific studies of dynamical systems, one of the most intriguing problems is the question of *reversibility*, or strictly speaking the problem of the time arrow, which is related to the statistical law of thermodynamic entropy.

  This means that the present moment is always separated from the future and of course from the past; consequently the world is historical. Basically, we all know that it is not possible to go back into the past. In particular, our biological clock is a special case of a thermodynamic clock. Unfortunately, when entropy achieves its maximum every complex organism will die. Therefore, *death* is not only a private tragic event, but may also be regarded as participation in the structure of the Cosmos.

The spiritual and moral evolution of every human of course depends on that person. Therefore, following the critical *rationalism* of Popper (see page 58), Heller has also noted that the decision to be rational in human life is a moral choice. Rationality then becomes a *morality of thinking*.

5.2 The Meaning of Life

Following the seminal works of the two Templeton Prize winners, Bernard d'Espagnat (2009) and Michał Heller (2008), we shall now consider the consequences of science and religion for the *meaning* of human life in the surrounding Universe (d'Espagnat, 1983; Heller, 1996, 2010).

Because every human being is a part of the Universe, the question of the meaning of humanity is strictly *related* to the meaning of the Universe. The existence of meaningless human life in a meaning Universe would be unlikely, for it would be logically inconsistent.

Naturally, in order to achieve happiness in our personal life, it is not enough to enjoy the present moment (as suggested by Father Józef M. Bocheński, OSB, 1902–1995): the Universe should rather have a *global* meaning, which is not limited to the given moment (Heller, 2010).

We should therefore teach ourselves how to live each present moment in our lives. On the other hand, however, because the appearance of awareness was a critical moment of human history, we can continuously ask ourselves about our own future and the final *objective*.

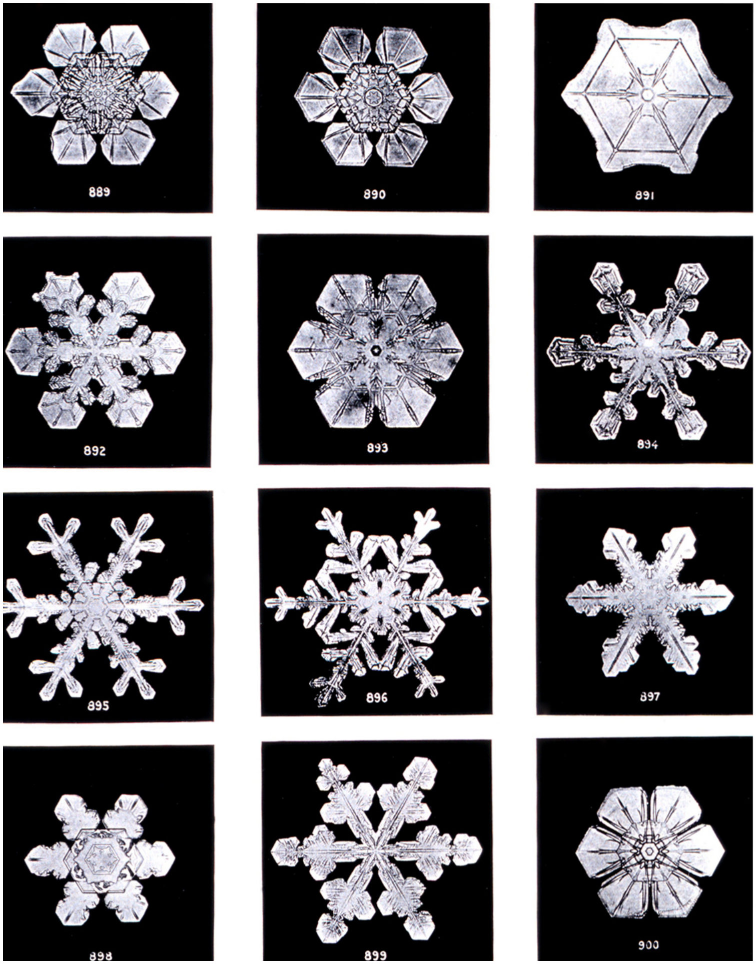

Figure 5.2 Snowflake patterns (credit: W. A. Bentley).

5.2.1 Life and Death in Nature

The origin of living matter is still a Big Mystery[2]. According to the second law of thermodynamics, in any isolated system composed of many particles approaching equilibrium, the total entropy always increases; consequently disorder must grow and natural processes on a macroscopic scale are irreversible in time. For example, after lighting a cigar, smoke will fill an entire room.

[2] https://wonderverse.home.blog/2019/04/21/life-and-death-in-nature/

Figure 5.3 A complex termite ‘cathedral’.

We know that living organisms, including our human body, are composed of a huge number of atoms and molecules. Because all processes are evolving according to a unidirectional arrow of time, and disorder should grow with time. Thus, from the thermodynamic point of view, the death of living creatures in nature is the consequence of increasing entropy and, in a certain sense, can be considered participation in the evolution of the whole Universe.

However, in spite of this fundamental law, we often observe the emergence of complex states far from thermodynamic equilibrium that are important for the origin of life, as briefly discussed on page 22. These structures often exhibit complex fractal shapes that can however be obtained by the recurrence of a given simple geometrical pattern, as thoroughly discussed in Section 3.3. One of many examples is a snowflake's fractal pattern obtained by the iteration of simple geometrical figures like a triangle, a square, or a hexagon, as illustrated in Figure 5.2, photographed by Wilson A. Bentley (1865–1931). A 'cathedral' mount is a natural complex system produced by termite colony that illustrates a classic example of emergence in nature generated by simple rules[3], as shown in Figure 5.3.

Based on my experience in the theory of physics, as discussed in this book, it would seem that, from the scientific point of view, coping with *death* is always hopeless. However, one should note that entropy is surely a thermodynamic quantity of many body systems and the asymmetry between past and future that results from irreversibility of time is only statistical in nature. Therefore, we can expect that the experience of passing away that results from the unidirectional arrow of time can only be attributed to complex bodies, and this does not necessarily apply to simple (non-complex) systems. Unexpectedly, nonlinear contemporary science, based on fractals and deterministic chaos theory, as explained in Section 3.3, give us many examples of where systems that look complicated could result from simple but nonlinear laws (Macek, 2000), see e.g., Figures 5.2 and 5.3. Therefore, if death is a consequence of *complexity*, then something that is not complex does not exist in any time flow and hence one can say that it cannot die.

Be that as it may, from a human, dramatic, and eschatological perspective, when expecting our own biological death, those who believe can look for hope in the rationality of God, who is the source of all natural and supernatural laws. In my own personal opinion and with the help of the modern philosophy of science, I am also quite convinced that the natural sciences can shed light on the religious belief that offers me the hope that death is a mysterious passage from time to ETERNITY.

[3] Taken from https://en.wikipedia.org/wiki/Emergence

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6
Epilogue

The main propose of this little book has been to look at the fundamental problem of protology in view of philosophy, science, and religion: to ponder the question: what is the origin of the World?, as briefly outlined in Chapter 1. I have wished to show the reader how it might be possible to reconcile the Biblical and modern scientific wordviews. Next, based on modern science, including the evolution of the Universe and quantum models, as discussed in Chapter 2, and of nonlinear dynamics and fractals in Chapter 3, it is now clear that there should be **no** contradiction between e.g.,

- evolution and creation
  (in favor of evolutionary creationism),
- determinism and indeterminism
  (in view of deterministic chaos theory and quantum mechanics).

In particular, in Section 1.1.3, I have proposed a novel hypothesis that the modern scientific theories of *nonlinear* dynamics, *chaos*, and *fractals* may play an important role in understanding the origin of the primordial world. Personally, I still believe that modern mathematical theories help us to understand the origin of the Universe (Macek, 2013). This is illustrated in Figure 6.1 (see also the cover of my book from 2010) from 13th century *Bible moralisée*, adopted by B. B. Mandelbrot (1924–2010), the Polish-born, French and American mathematician, who coined the term *fractal*, see Section 3.3, in order to understand the fractal geometry of nature, including the primordial world. Therefore, it is not surprising that the same icon is presently used by both theologians and scientists working in disciplines of the mathematical natural sciences.

Finally, following Chapter 4 on science and religion and the final Chapter 5 on the problems of the meaning of the world and life, it is important to note the following points, ones that are altogether convincing for me:

- I believe that the modern concept of the theology of science indeed can BRIDGE SCIENCE AND RELIGION, thus giving *meaning* to life (Macek, 2010, 2011).
- Hence, it would seem that both science and religion provide important contributions that shape our lives, as we experience the world in which we are immersed.
- I also argue that if we do not wish to continue theological studies in separation from science, then the philosophy of science and classical theology should adopt into their thinking the most important ideas and achievements of the mathematical natural *sciences*.

Therefore, we all hope that this will permit a better understanding of humans in their relation to both the Universe and the transcendent *Reality*.

Figure 6.1 Christ Pantocrator.

The icon of Christ Pantocrator painted by Julita Jaśkiewicz-Macek was originally taken from the *Bible moralisée* from the period 1220–1230, at present in the National Austrian Library in Vienna (codex 2554). The legend in the French dialect of the Eastern Champagne was:

Ici crie Dex ciel et terre
soleil et lune et toz elemenz.

(Here creates God sky and earth
sun and moon and all elements.)

(Mandelbrot, 1982, Plate C1) transcription reads:
Here God creates circles, waves, and fractals.

Appendix A
The Standard Model

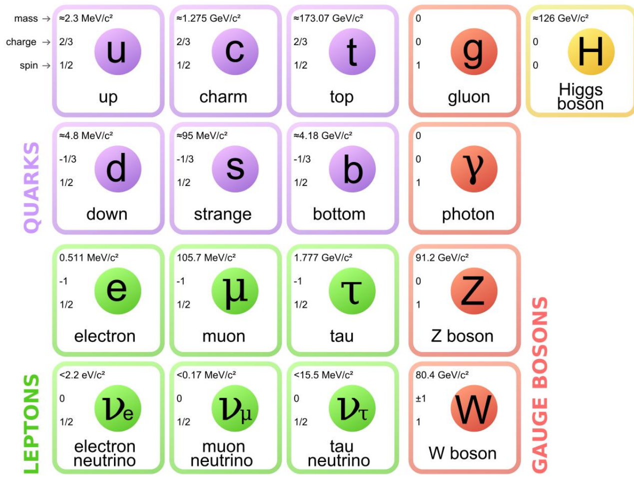

Figure A.1 Three generations of particles, with gauge bosons in the fourth column and the Higgs boson in the fifth.

From https://en.wikipedia.org/wiki/Standard_Model

The Standard Model of particle physics is the theory describing three of the four known fundamental forces (the electromagnetic, weak, and strong interactions, and not including the gravitational force) in the Universe, as well as classifying all known elementary particles. It was developed in stages throughout the latter half of the 20th century, through the work of many scientists around the world, with the current formulation being finalized in the mid-1970s upon experimental confirmation of the existence of quarks. Since then, confirmation of the top quark (1995), the tau neutrino (2000), and the Higgs boson (2012) have added further credence to the Standard Model. In addition, the Standard Model has predicted various properties of weak neutral currents and the W and Z bosons with great accuracy.

Appendix B
The Structure and History of the Universe

The illustration in Figure B.1 exhibits the structure of the Universe within 1 billions light years of Earth, showing local superclusters (approximately 63 million galaxies are shown here), while a diagram of Earth's location in the observable Universe and neighbouring superclusters of galaxies[1] are depicted in Figure B.2.

This illustration of the location of Earth in the Universe consists of a series of nine frames that show: 1. The Earth and its natural satellite, the moon. 2. Earth's location in the inner solar system in relation to other planets and their orbits (distance measures are average distances of objects to the Sun, not the Earth). 3. The Sun's location in the solar system with respect to the outer solar system and its planetary bodies. 4. The solar system's position amongst the closest stars in the Milky Way. 5. Its location in the Orion arm of the Milky Way Galaxy. 6. The Milky Way Galaxy's location in the Local Group, its nearest major and minor galaxies. 7. The Local Group's location within the Virgo Supercluster as part of the larger Laniakea Supercluster. 8. That supercluster's location amongst the nearest superclusters in the Universe. 9. The observable universe with the local superclusters barely visible in the center. Each frame contains annotations with the names of astronomical objects (in white) and their distance from Earth (in purple). The blue text indicates the object found in the previous frame. Some items have no distance attached while others are rough approximations or averages due to variances in research findings. Many of the objects and visible distances are inaccurate as they have been increased substantially for illustration purposes. The purple text indicates the real astronomical distances in several units: kilometers, light years (one light year is about 9.5 trillion kilometers, or 5.9 trillion miles), kilolight-years (equal to 1,000 light years), and megalight-years (equal to 1,000,000 light years).

[1] Taken from:
https://en.wikipedia.org/wiki/Supercluster#/media/File:Superclusters_atlasoftheuniverse.gif
https://en.wikipedia.org/wiki/File:Location_of_Earth_(3x3-English_Annot-small).png

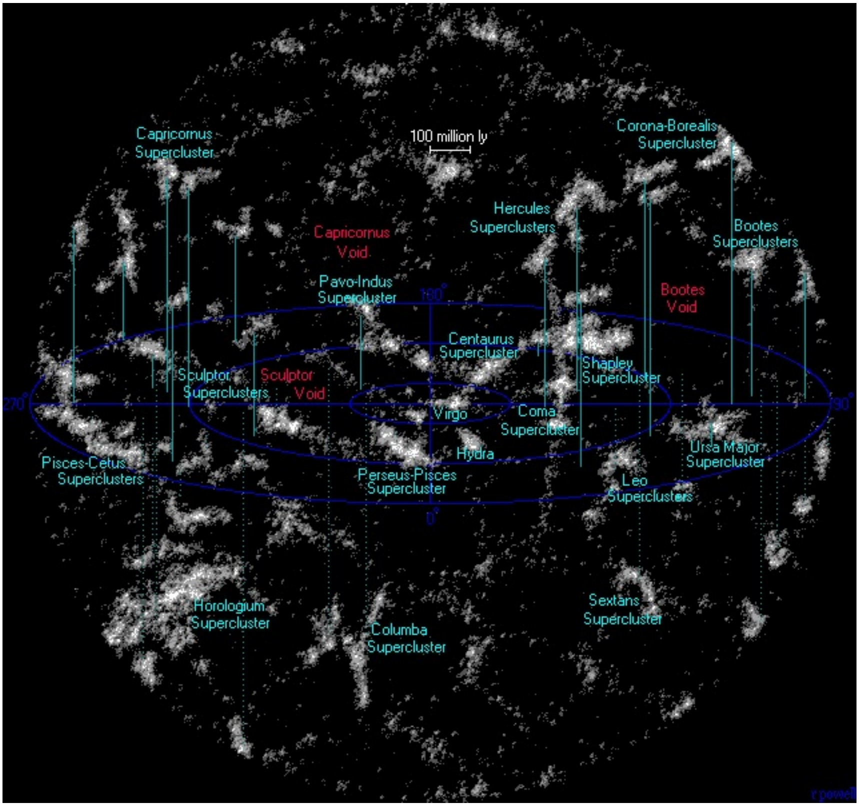

Figure B.1 A map of the Superclusters and voids nearest to Earth (credit: R. Powell, NASA).

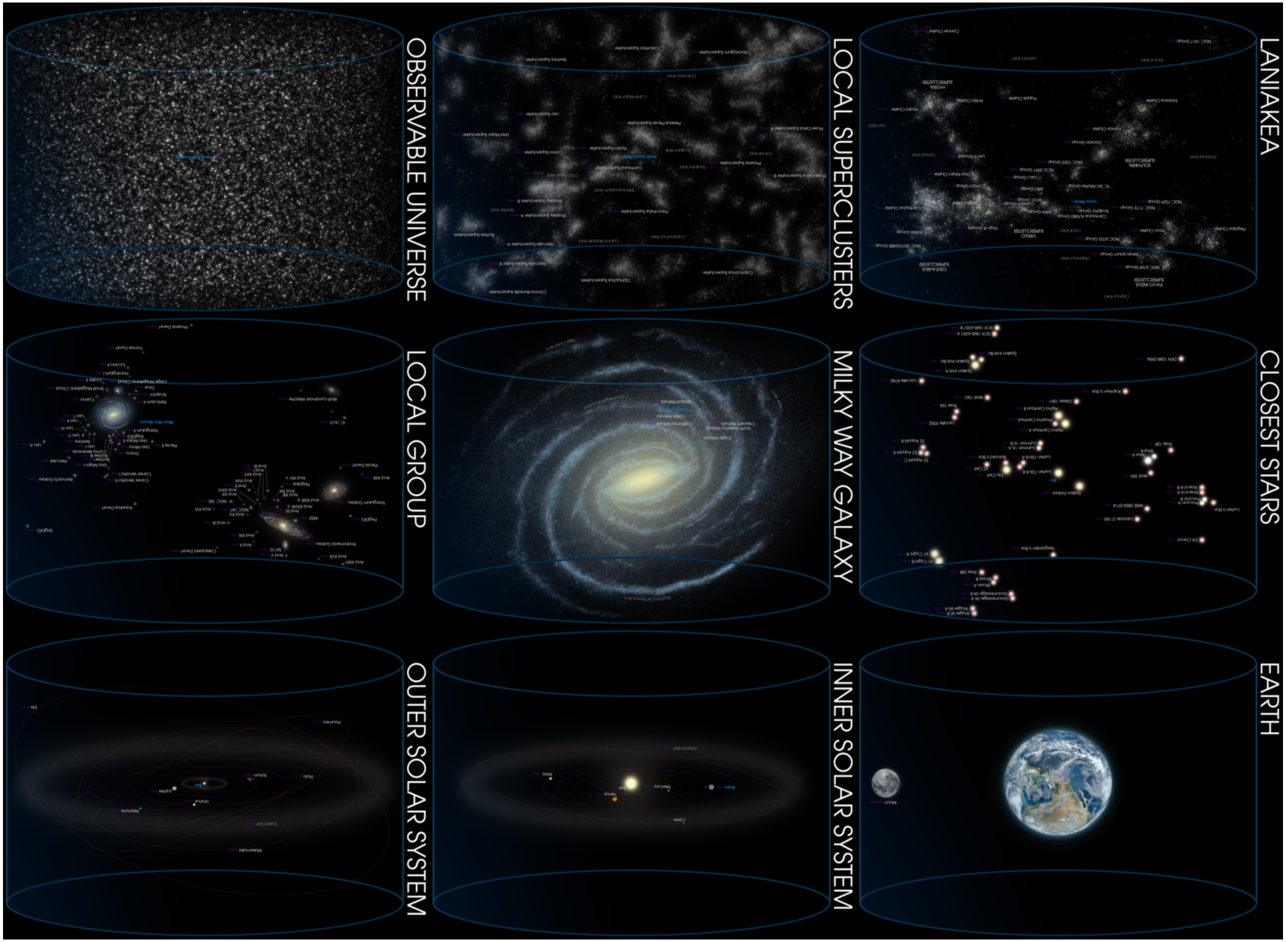

Figure B.2 The structure of the Universe (credit: A. Z. Colvin NASA).

The Story of Our Universe[2]

This illustration summarizes the almost 14-billion-year-long history of our Universe. It shows the main events that occurred between the initial phase of the cosmos — where its properties were almost uniform and punctuated only by tiny fluctuations — to the rich variety of cosmic structure that we observe today, ranging from stars and planets to galaxies and galaxy clusters.

The Planck mission has made the most precise map ever of the oldest light from our Universe, the cosmic microwave background, harking back to less than 400,000 years after the big bang. Patterns of light in this map reflect not only events that happened just moments after the big bang, but also the light's long journey from the distant Universe to Earth. By studying these patterns, scientists can learn about the origins, fate and ingredients of our Universe.

Planck is a European Space Agency mission, with significant participation from NASA. NASA's Planck Project Office is based at NASA's Jet Propulsion Laboratory, Pasadena, Calif. JPL contributed mission-enabling technology for both of Planck's science instruments. European, Canadian and U.S. Planck scientists work together to analyze the Planck data.

[2] Taken from the NASA website
https://www.nasa.gov/mission_pages/planck/multimedia/pia16876b.html#.W4-3s16Lnq0

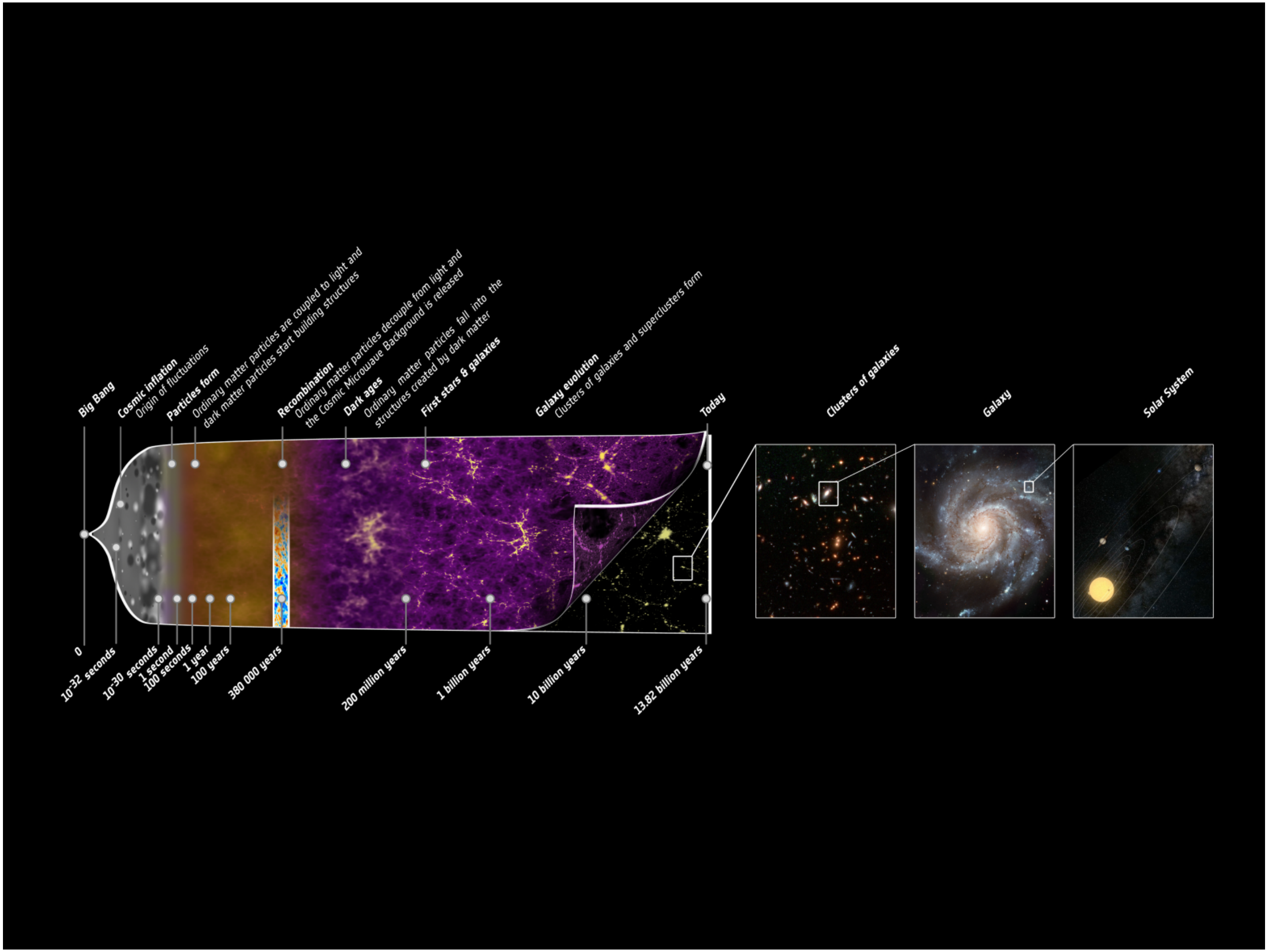

Figure B.3 The history of the Universe (credit: NASA).

Acknowledgments

The main idea for the origin of the Universe as based on nonlinear dynamics and chaos was presented during my invited talk at a conference held in Athens in 2013 (unpublished abstract) and at the conference entitled *Do Emotions Shape The World?* held in Assisi in 2014. That presentation was included in the Biennial Yearbook of the European Society for Study of Science and Theology (ESSSAT). Benoit Mandelbrot coined the concept of fractals and I enjoyed meeting him in Nice twenty-five years ago. Steven Strogatz provided me with the exercises to his book on nonlinear dynamics and chaos, which were useful during my lectures at Faculty of Mathematics and Natural Sciences of the Cardinal Stefan Wyszyński University (UKSW) in Warsaw and at various university and scientific institutions (Calabria, San José dos Campos, and Brussels). My research studies on chaos and turbulence in fluids and space plasmas have been supported by the National Science Center (NCN) in Poland, recently through grant 2014/15/B/ST9/04782. I would like to thank the NASA/WMAP Science and ESA/NASA Planck Teams for providing the schematic of the Evolution of the Universe and the illustrations of the history of the world. I am also grateful for the schematic of the Standard Model of elementary particles that is available from the website of CERN. I especially benefited from a daily support of my wife, who painted the icon of Christ Pantocrator resulting from Biblical inspiration.

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Glossary

attractor	an invariant set in phase space toward which a time history evolves.
bifurcation	a change in the dynamical behavior of a system when a control parameter is varied.
black hole	a compact mass in curved spacetime preventing any signal from escaping it.
chaos	nonperiodic behavior exhibiting sensivity to initial conditions.
cosmological model	a standard model of the evolution of the Universe after the Big Bang.
cyclic cosmological model	a non-standard cosmological scenario with an infinite sequence of big bangs and crunches.
elementary particle model	a standard model of strong and electroweak point-like interactions between elementary particles.
ex nihilo	a concept of the creation of the world out of nothingness.
fractal	fragmented shape. Usually self-similar.
gravitational waves	disturbances in the curvature of spacetime generated by an accelerated mass.

map	a mathematical relation for discrete time evolution.
multifractal	a set of itertwined fractals.
noncommutative geometry	an abstract algebra of functions and operators where the multiplication is not reversible.
phase space	a coordinate space defined by the state variables of a dynamical system.
protology	a basic science of the origin of everything.
quantum gravity	a quantum theory of gravitation on primordial extremely small scales.
string theory	theory of string-shaped interactions between elementary particles.

Author index

Subject index